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Many students have difficulties in algebra and other types of mathematics because they lack proficiency in basic arithmetic skills. In Chapter 1, there is a review of these fundamental skills. **Mastery of Section 1.1 will be especially important in providing students with the requisite skills for being successful in the algebra portion of the course.**

Course Outcomes:

- Perform and execute basic arithmetic operations and simplify expressions involving exponents and square roots
- Recognize and apply mathematical concepts to real-world situations

1.1 Signed Numbers and Order of Operations

Students will learn about different types of numbers, absolute value, order of numbers, and how to perform basic arithmetic operations with integers. Exponents and various properties of real numbers are introduced. Mastering this section will give students important skills for the rest of the course.

1.2 Prime Factorization, GCF, LCM

Exponents, factors, prime factors, common factors, multiples, common multiples, and applications are covered. Students are introduced to these topics that are important when working with fractions and some applied problems.

1.3 Fractions

Rational numbers, proper fractions, improper fractions, mixed numbers, equivalent fractions, and simplest form are defined and used. Students learn to do arithmetic with fractions including with order of operations.

1.4 Square Roots

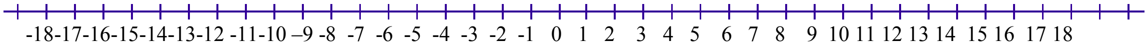
Square roots and irrational numbers are defined. Students learn to do arithmetic operations and applications with square roots.

1.5 Exponents and Scientific Notation

The definition of exponents is reviewed. The rules for exponents are presented and students learn to use them. Scientific Notation is presented and the rules of exponents are used to multiply and divide.

Perhaps the most typical notion of a number is what we do when we are counting. We start at 1 and continue: 1,2,3,4,5,.. These are called **natural numbers.** By adding zero to these natural numbers we get the **whole numbers,** which can be expressed as a set {0,1,2,3,4,5,…}. By now we are familiar with the arithmetic operations of addition, subtraction, multiplication, and division with whole numbers.

Integers include the whole numbers and their opposites or negatives. Fractions and decimals are not integers. When we start talking about arithmetic with integers (signed numbers both positive and negative) things get a little tricky. Even if students are comfortable with arithmetic and these signed numbers, they often make careless mistakes with negative signs. So, we will look at integers more closely. Integers can be listed on a number line as follows.



Remember: Numbers to the left on the number line are smaller than numbers to the right.

Statement	Symbolically
Negative four is less than six	– 4 < 6
Negative two is greater than negative seven	–2 > –7
Three is greater than negative twelve	3 > –12

The absolute value of an integer is the distance to zero on the number line. Because distance is a positive value, we often just keep the integer positive (if it is positive) or make it positive (if it is negative). We use two vertical lines to indicate absolute value.

Examples

1. Evaluate $|-3|$. The two vertical lines indicate the absolute value of –3.

$$|-3| = 3$$

Make the number positive and that is it. Note that the –3 is three units from zero on the number line.

2. Evaluate $|5|$. The two vertical lines indicate the absolute value of 5.

$$|5| = 5$$

Keep the number positive and that is it. Note that the 5 is five units from zero on the number line.

3. Write $-3, |-7|, 2, -|-9|, -(-3), 12$ in order from smallest to largest.

Steps	Reason
$-3, \|-7\|, 2, -\|-9\|, -(-3), 12$	Simplify the numbers and write them below.
–3, 7, 2, –9, 3, 12	
– 9, –3, 2, 3, 7, 12	Write the simplified numbers in order from smallest to largest by looking at a number line if needed. Write the original form in that same order.
$-\|-9\|, -3, 2, -(-3), \|-7\|, 12$	

Rules for adding and subtracting integers:

Steps for adding two integers (Integers are positive or negative numbers with no fractional part):

Same sign (both numbers are positive or both are negative)
Size: add absolute values (add as if both numbers are positive)
Sign: same sign as addends (numbers being added)

Different Signs (one negative another positive number are added)
Size: subtract the absolute values
Sign: same sign as the number with larger absolute value

We should ask ourselves where these rules come from? Do they make sense? Adding two positive numbers is familiar. When we add two negative numbers we can take the example of adding two debts. If somebody owes one friend \$5 and another friend \$6, then the person owes \$11. So, - 5 + (-6) = -11.

For adding integers with different signs, we can think of money as well. Somebody that has \$5 but owes \$3 really has \$2, which is to say 5 + (-3) = 2. However, if somebody has \$5 and owes \$8 then they have a negative \$3. This would be the same as 5 – 8 = -3 or 5 + (-8) = -3. Once we accept the rules we need to apply them with confidence.

Examples

4. Add: –15 + 6

Steps	Reason
–15 + 6	Adding one positive and one negative integer
–9	Size: Subtract the absolute values. 15 – 6 = 9 Sign: Because 15 is larger the sign is negative.

5. Add: –9 + (–14)

Steps	Reason
–9 + (–14)	Adding two negative integers. (Same sign)
–23	Size: Add the absolute values. 9 + 14 = 23 Sign: Negative for adding two negative numbers.

Steps for **subtracting two integers:**

Subtraction can be rewritten as adding the opposite of a number. So,

a – b = a + (–b)

Thinking of subtraction as adding the opposite lets us use the rules for adding two integers.

Examples

6. Subtract: 7 – 15

Steps	Reason
7 – 15	If we cannot subtract in one step, rewriting the problem lets us discuss the steps.
7 + (–15)	Rewrite subtracting 15 as adding –15
–8	Size: Take the difference of (subtract) the absolute values. Sign: Since 15 is larger, the sign is negative.

7. Subtract: 18 – 11

Steps	Reason
18 – 11 = 7	Just do the subtraction. It would confuse things to rewrite this problem.

8. Subtract: –7– 11

Steps	Reason
–7– 11	If you cannot do it in one step, rewriting the problem lets us discuss the steps.
–7+ (–11)	Write subtracting 11 as adding –11.
–18	Size: Take the sum of (add) the absolute values. 7 + 11 = 18 Sign: Since both are negative, sign is negative.

The above three examples can be done in one step. By thinking of the subtraction as adding the opposite, we can apply the rules for adding signed numbers.

When we combine subtraction with negative numbers sometimes the symbols look awkward. Never fear, we can simplify:

Statement	Symbols	Simplified
The opposite of negative eight	– (–8)	8
Twelve minus negative five	12– (–5)	12 + 5
9 plus negative seven	9 + (–7)	9 – 7

Making the opposite of a negative number positive and changing minus a negative number into addition can really help to simplify some otherwise complex expressions.

Example

9. Simplify 11 – (–2) – 18 + 4

Steps	Reason
11 – (–2) – 18 + 4	Write the original problem.
11 + 2 – 18 + 4	Take a step to change “minus a negative” to “plus”.
13 – 18 + 4	Add and subtract from left to right.
–5 + 4	13 – 18 is the same as 13 + (–18). Take the difference 18 – 13 = 5 and the sign is negative.
–1	For –5 + 4, take the difference 5 – 4 and the sign is negative.

Rules for multiplying and dividing integers:

Steps for **multiplying two integers:**

Size: multiply the absolute values (ignore negative signs) of the factors

Sign:

- Both positive factors yield positive product: 2 · 3 = 6
- Both negative factors yield a positive product: (–4)(–6) = 24
- One negative factor and one positive factor yield a negative product: (–3)·5 = –15

Steps for **dividing two integers:**

Same steps as multiplication except do division.

36 ÷ (–4) = –9	Divide as if the numbers are positive. One number negative and another positive yields a negative quotient. (Quotient is the result of division.)
(–40)÷(–8) = 5	Divide as if the numbers are positive. Because both numbers are negative, the quotient is positive.

Examples

10. Evaluate: $-72 \div (-9)$

Steps	Reason
$-72 \div (-9) = 8$	72 ÷9 is 8. Because both -72 and -9 are negative, the quotient is positive.

11. Evaluate –(–3)(–2)(–7).

Steps	Reason
–(–3)(–2)(–7)	Multiply the numbers as if they were positive. Because there are four negative signs the product is positive.
42	

12. Find the quotient of –92 and 4.

Steps	Reason
–92 ÷ 4	Rewrite quotient as division.
–23	92 ÷ 4 is 23. Because one of the numbers is negative and the other is positive, the quotient is negative.

The Multiplication Property of Zero:

A · 0 = 0 or 0 · A = 0

Multiply any number by zero and get zero. For example, 3 · 0 = 0.

The Multiplication Property of One:

$A \cdot 1 = A$ or $1 \cdot A = A$

Multiply any number by one and get the same number. $1 \cdot 7 = 7$ We can call 1 the identity of multiplication because it gives us what we started with when we multiply.

The Commutative Property of Multiplication:

$A \cdot B = B \cdot A$

We can multiply in either order. Both $5 \cdot 3$ and $3 \cdot 5$ are 15

The Associative Property of Multiplication:

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

If there is only multiplication, the numbers can be grouped either way.

$$\begin{aligned} &(4 \cdot 2) \cdot 5 \\ =\ & 8 \cdot 5 \\ =\ & 40 \end{aligned} \qquad\qquad \begin{aligned} &4 \cdot (2 \cdot 5) \\ =\ & 4 \cdot 10 \\ =\ & 40 \end{aligned}$$

Both ways we get 40.

Using the Commutative Property and the Associative property, any list of integers that are multiplies can be multiplied in any order.

Division properties

$\frac{0}{a} = 0$, Zero divided by any number is zero. $\frac{0}{12} = 0$

$\frac{a}{a} = 1$, Any number divided by itself is one. $\frac{9}{9} = 1$

$\frac{a}{1} = a$, Any number divided by one is itself. $\frac{6}{1} = 6$

$\frac{a}{0}$ is undefined. Never divide by zero. It is impossible. $\frac{5}{0}$ is undefined.

The Addition Property of Zero:

A + 0 = A or 0 + A = A
In other words, if you add zero you get what you started with. 5 + 0 = 5

We can call zero the identity for addition because we get what we started with when we add 0.

The Commutative Property of Addition:

A + B = B + A

We can add in either order. Both 5 + 3 and 3 + 5 are 8

The Associative Property of Addition:

(A + B) + C = A + (B + C)

If there is only addition the numbers can be grouped either way.

	(4 + 2) + 5		4 + (2 + 5)
=	6 + 5	=	4 + 7
=	11	=	11

Both ways we get 11.

The Inverse Property of Addition

A + (–A) = 0 or –A + A = 0

If a number is added to its opposite, the sum is 0. For example –5 + 5 = 0.

Using the Commutative Property and the Associative property, any list of integers to be added can be added in any order. For example,

$(-2)+9+12+1=9+1+12+(-2)$ We can change the order by the commutative property to add in any order that we like, which makes it easier to add here.

Being able to add or multiply in any order will be helpful in algebra. So remember:
If there is a list of numbers that is added together we can add in any order.
If there is a list of numbers multiplied together we can multiply in any order.

Exponents can be used to write repeated multiplication. $5^3 = 5 \cdot 5 \cdot 5$ The exponent of 3 tells us how many times to multiply the base, which is 5.

Examples

13. Find 2^4

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$

14. Find $(-3)^2$

$$(-3)^2 = (-3)(-3) = 9$$

15. Find -4^2

$-4^2 = -4 \cdot 4 = -16$ This case catches many people. The trick is that the exponent only goes with the symbol in front. So, here the exponent only goes with the 4 and the negative sign means that 4^2 will be negative. If we want a negative number to be squared, then we need parentheses like the example with $(-3)^2$.

The **order of operations** is fundamental to most problems in algebra. We need to know them and be able to use them comfortably. The **order of operations** is as follows:

1. **Parentheses** inside to outside.
2. **Exponents.**
3. **Multiplication** and **division** together as they appear from left to right.
4. **Addition** and **subtraction** together as they appear from left to right.

Some people remember **PEMDAS** to help keep the order straight. Back in the day you may have heard Please Excuse My Dear Aunt Sally as a way to remember PEMDAS. Eventually, we want to be completely comfortable with the above order of operations. Be careful! Division and multiplication are at the same level. If the division comes before multiplication as we read left to right, then we do that division first. Addition and subtraction are similar in this. If subtraction is to the left of addition with no other symbols or operations, we do the subtraction before the addition.

Examples

16. Simplify: $4 + 3(5^2 - 15)$

Steps	Reasons
$4 + 3(5^2 - 15)$	Think about the operations that appear and their order.
$4 + 3(25 - 15)$	
$4 + 3(10)$	1. Work within parentheses. Evaluate the exponent before the subtraction.
$4 + 30$	2. Keep working within the parentheses by doing the subtraction.
34	3. Do multiplication before addition. 4. Add.

17. Simplify: $(-6)^2 \div 4 + 5 \cdot (-2)$

Steps	Reasons
$(-6)^2 \div 4 + 5 \cdot (-2)$	Look at the operations: exponents, division, addition, and multiplication. Do exponents.
$36 \div 4 + 5 \cdot (-2)$	Do division and multiplication at the same time.
$9 + (-10)$	Adding signed numbers
-1	

To check this problem you need to redo it on a separate piece of paper.

18. Simplify $-12(6-8)+1^3 \cdot 3^2 \cdot 2 - 2(-12)$

Steps	Reasons
$-12(6-8)+1^3 \cdot 3^2 \cdot 2 - 2(-12)$	Look at the operations: multiplication, subtraction, addition, and exponents. Work inside parentheses.
$-12(-2)+1^3 \cdot 3^2 \cdot 2 - 2(-12)$	
$-12(-2)+1 \cdot 9 \cdot 2 - 2(-12)$	Do exponents: $1^3 = 1$ and $3^2 = 3 \cdot 3 = 9$
$24 + 18 + 24$	Multiply: $-12(-2) = +24$ $1 \cdot 9 \cdot 2 = 18$ $-2(-12) = +24$
66	Add

To check this problem you need to redo it on a separate piece of paper.

Note: In each step one operation is being performed and everything else is being rewritten. Always rewrite everything you intend to simplify in later steps. Some operations can be performed slightly out of order. The parentheses and the exponents could have been done together in the last problem because addition separates them, but be careful.

19. Evaluate: $7-3\left[1-\left(2-\left(-3\right)\right)^2\right]$

Steps	Reasons
$7-3\left[1-\left(2-\left(-3\right)\right)^2\right]$	Write minus a negative as plus first because it is inside the parentheses.
$7-3\left[1-\left(2+3\right)^2\right]$	Then add, 2+3=5
$7-3\left[1-5^2\right]$	Square the 5 before subtraction.
$7-3\left[1-25\right]$	Subtract in the brackets.
$7-3\left[-24\right]$	Multiply before subtracting. -3 times -24 is +72
$7+72$	Add to get the answer
79	

Signed numbers and the order of operations are very important throughout all of mathematics. The order of operations gives the math world a structure to follow and signed numbers always need attention since many careless errors involve negative signs.

Exercises:

Place the correct symbol > (greater than), < (less than), or = (equal to) in the ☐ between the two numbers:

1. $7 \square 9$

2. $5 \square -4$

3. $-8 \square -12$

4. $-9 \square 0$

5. $-4 \square -3$

6. $|-3| \square 3$

7. $12 \square |-15|$

8. $|-5| \square 11$

9. $|-6| \square |4|$

Order from smallest to largest:

10. $|-8|, -4, 0, 6, |3|, |-10|$

11. $7, |-11|, -1, 3, |-2|, |0|$

12. $|-3|, -9, 6, |12|, 0, |-14|$

13. $-2, 1, |-6|, -5, 0, |11|, |-7|$

Simplify without using a calculator:

14. $-3 + 8$

15. $5 + (-7)$

16. $-12 + 5$

17. $-15 + (-6)$

18. $-8 + (-4)$

19. $18 + (-14)$

20. $-19 + (-16)$

21. $-25 + (-17)$

22. $-12 - 5$

23. $15 - 22$

24. $-3 - (-11)$

25. $-20 - (-11)$

26. $-10 - 12$

27. $12 - 24$

28. $-8 - (-17)$

29. $-14 - (-9)$

30. $-6 \cdot (-9)$

31. $5 \cdot (-9)$

32. $-8 \cdot (-7)$

33. $-4 \cdot 10$

34. $9 \cdot (-3)$

35. $-7 \cdot 9$

36. $-9 \cdot (-8)$

37. $81 \div (-9)$

38. $35 \div (-7)$

39. $-72 \div 8$

40. $-64 \div (-8)$

41. $-49 \div 7$

42. $-63 \div (-7)$

43. $100 \div (-5)$

44. $-36 \div (-4)$

45. 5^3

46. 6^2

47. $(-4)^2$

48. -2^3

49. $(-8)^2$

50. -3^2

51. -2^4

52. $(-7)^2$

53. $2 + 5(3)$

54. $15 - 3 \cdot 4$

55. $3 + 18 \div (-3)$

56. $-5 - (-12) \div (-4)$

57. $4 - (-21) \div (-3)$

58. $7 + 2(10 - 8)$

59. $-3 - 5(3 + 2)$

60. $8 - 5(3 - 7)$

61. $-10 - 4(3 - 7)$

62. $(5 - 2^2) \cdot (3^2 - 6)$

63. $(3 - 4^2) - (2^3 - 10)$

64. $8 - 4[3(5 - 2) - (2 - 4)]$

65. $3 + 2[4(2 - 3^2) - 2(5 - 4^2)]$

66. $6 + 5[2(5 - 2^3) + 3(-20 + 5^2)]$

67. $12 - 3[3(8 - 2^3) - 2(15 - 3^2)]$

68. $3(5{-}2^2) - 4(3^2 - 12)$

69. $3\{5 - 2[-5 - (-8)] - 3[-2^2 + 4(5 - 2)]\}$

70. $-2\{3[5 + (-7)] - 4[-2^4 + 2(-3)^2]\}$

71. $-4\{2[-3 + (-7)] - 4[2^3 + 4(-2)^3]\}$

For 72 and 73, remember that when we borrow or spend money we can use negative numbers and when we receive money we can use positive numbers.

72. Jack starts the weekend with $20. He then borrows $7 from each of six different friends and spends the money immediately. Another friend pays Jack back $15. Jack also spends $40 over the weekend. How much money does Jack have at the end of the weekend?

73. Maria started the month with $800. Each of the four weeks she spent $230 on household expenses. For the month she spent $275 on entertainment and eating out. How much did she have to borrow from her roommate to make it to the end of the month?

74. A diver goes down 20 feet in the first 5 minutes, 25 feet the second five minutes, and then 11 feet the next five minutes to reach the bottom. When the diver comes up he moves 8 feet every 5 minutes and he stops for 3 minutes every 8 feet. How long does it take the diver to go down? How long does it take the diver to return to the surface?

75. Rome was founded in 753 BC. The United States declared independence in 1776 AD. Use the appropriate negative number with subtraction to find the difference in time between the founding of Rome and the Declaration of Independence. Hint: Do not forget to take into account that there is no year 0 since we went from 1 BC to 1 AD.

Exponents can be used to write repeated multiplication. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ The exponent of 4 tells us how many times to multiply the base, which is 3.

Factors are multiplied to get the **product**. Since 3·5=15, 3 and 5 are factors of 15. Also, 1 and 15 are factors of 15 because 1·15=15. In the end 1,3,5,15 are all factors of 15.

Prime numbers have only two whole number factors which are 1 and the number itself. Examples of prime numbers are 2,3,5,7,11,13,17,19,23,29,31,…

We can write the **prime factorization** of a number by dividing the number by prime factors repeatedly.

Example

1. Find the prime factorization of 150.

$$\begin{array}{r} 75 \\ 2\overline{)150} \end{array}$$

Begin by dividing by the smallest prime number 2.

$$\begin{array}{r} 25 \\ 3\overline{)75} \\ 2\overline{)150} \end{array}$$

Divide by the next prime number 3.

$$\begin{array}{r} 5 \\ 5\overline{)25} \\ 3\ \overline{)75} \\ 2\ \overline{)150} \end{array}$$

Divide by the next prime number 5.

When the last factor is prime, we stop. The prime factorization is the product of the factors (or divisors) 2,3,5, and the remaining prime factor 5.

Since the 5 is a factor twice, the prime factorization is $2 \cdot 3 \cdot 5^2$.

Note: 2,3, and 5 are all prime numbers and the product $2 \cdot 3 \cdot 5^2 = 2 \cdot 3 \cdot 5 \cdot 5$ does equal 150.

The **multiples** of a number have the number as a factor.

Multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, …
Multiples of 9 are: 9, 18, 27, 36, 45, 54, 63, 72, 81,…

Common Multiples are multiples of two or more numbers.

The common multiples of 6 and 9 are 18, 36, 54, 72, …

The **Least Common Multiple (LCM)** is the smallest common multiple. The least common multiple of 6 and 9 is 18.
Note: The least common multiple of 6 and 9 is always at least as big as the larger of the numbers. 18 is a multiple of 6 because 6 · 3 = 18, and 18 is a multiple of 9 because 9 · 2 = 18.

To find the Least Common Multiple we can look at the multiples of the larger number until we find a multiple of the smaller number. We should also be able to find the LCM by first taking the prime factorization.

Finding the LCM by using the prime factorization:
1. First find the prime factorization as above.
2. Write all prime factors once and ignore the exponent.
3. Go back and chose the **largest** exponent for each of the prime factors.

Examples

2. Find the least common multiple of 36 and 30 by first finding the prime factorizations.

Steps	Reasons
3 $3\overline{)9}$ $2\overline{)18}$ $2\overline{)36}$	Prime factorization of 36: Divide by 36 by 2 because 2 is the smallest prime factor. Keep dividing by the smallest prime factor until the remainder is prime.
$36 = 2^2 \cdot 3^2$	The divisors and remainder make the prime factorization of 36.
5 $3\overline{)15}$ $2\overline{)30}$	Prime factorization of 30: Divide 30 by 2 because 2 is the smallest prime factor. Keep dividing by the smallest prime factor until the remainder is prime.
$30 = 2 \cdot 3 \cdot 5$	The divisors and remainder make the prime factorization of 30.
$2 \cdot 3 \cdot 5$	Write all prime factors once and ignore the exponent.
$2^2 \cdot 3^2 \cdot 5$	The **largest** exponents for the 2 and 3 are two from the prime factorization of 36
180	Multiply $2^2 \cdot 3^2 \cdot 5$. So, 180 is the LCM of 36 and 30.

3. Find the least common multiple of 8, 12, and 27 by first finding the prime factorizations.

Steps	Reasons
$\begin{array}{r} 2 \\ 2\overline{)4} \\ 2\overline{)8} \end{array}$	Prime factorization of 8: Divide 8 by 2 because it is the smallest prime factor. Keep dividing by the smallest prime factor until the remainder is prime.
$8 = 2^3$	The divisors and remainder make the prime factorization of 8.
$\begin{array}{r} 3 \\ 2\overline{)6} \\ 2\overline{)12} \end{array}$	Prime factorization of 12: Divide 12 by 2 because it is the smallest prime factor. Keep dividing by the smallest prime factor until the remainder is prime.
$12 = 2^2 \times 3$	The divisors and remainder make the prime factorization of 12.
$\begin{array}{r} 3 \\ 3\overline{)9} \\ 3\overline{)27} \end{array}$	Prime factorization of 27: Divide 27 by 3 because it is the smallest prime factor. Keep dividing by the smallest prime factor until the remainder is prime.
$27 = 3^3$	That is the prime factorization of 27.
$2 \cdot 3$	Write all prime factors once and ignore the exponent.
$2^3 \cdot 3^3$	The **largest** exponent for the 2 is the three from $2^3 = 8$ and the **largest** exponent for the 3 is the three from $3^3 = 27$.
216	Multiply $2^3 \cdot 3^3$. So, 216 is the LCM of 8, 12 , and 27.

Factors of a number divide the number evenly.
The factors of 12 are 1, 2, 3, 4, 6, and 12.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30

The **common factors** of 12 and 30 are 1, 2, 3, and 6.

The **Greatest Common Factor (GCF)** of 12 and 30 is 6.

Careful, students tend to confuse Least Common Multiple and Greatest Common Factor. The Greatest Common Factor is always smaller than the numbers because it is a common factor for the numbers. The Least Common Multiple is always larger than the numbers because it is a common multiple of the numbers.

Finding the GCF by using the prime factorization:

1. First find the prime factorization as above
2. Write all the **common** prime factors once and ignore the exponent.
3. Go back and chose the **smallest** exponent for each of the prime factors.

The only change is that now we use the smallest exponent for each factor.

Example:

4. Find the greatest common factor of 24, 48, and 72 by first finding the prime factorizations.

Steps	Reasons
$\begin{array}{r} 3 \\ 2\overline{)6} \\ 2\overline{)12} \\ 2\overline{)24} \end{array}$	Prime factorization of 24: Divide 24 by 2 because 2 is the smallest prime factor. Keep dividing by the smallest prime factor until the remainder is prime.
$24 = 2^3 \cdot 3$	The divisors and remainder make the prime factorization of 24.
$\begin{array}{r} 3 \\ 2\overline{)6} \\ 2\overline{)12} \\ 2\overline{)24} \\ 2\overline{)48} \end{array}$	Prime factorization of 48: Divide 48 by 2 because it is the smallest prime factor. Keep dividing by the smallest prime factor until the remainder is prime.
$48 = 2^4 \cdot 3$	That is the prime factorization of 48.
$\begin{array}{r} 3 \\ 3\overline{)9} \\ 2\overline{)18} \\ 2\overline{)36} \\ 2\overline{)72} \end{array}$	Prime factorization of 72: Divide 72 by 2 because it is the smallest prime factor. Keep dividing by the smallest prime factor until the remainder is prime.
$72 = 2^3 \cdot 3^2$	That is the prime factorization of 72.
$2 \cdot 3$	Write all common prime factors once and ignore the exponent.
$2^3 \cdot 3$	The **smallest** exponent for the 2 is the three from the 24 and the 72. The **smallest** exponent for the 3 is one from the 24 and 48.
24	Multiply $2^3 \cdot 3$. So, 24 is the GCF of 24, 48, and 72.

Note: 24, 48, and 72 all have 24 as a factor.

Exercises:

1. Find the prime factorization of 36.
2. Find the prime factorization of 45.
3. Find the prime factorization of 100.
4. Find the prime factorization of 360.
5. Find the prime factorization of 220.
6. Find the prime factorization of 720.
7. Find the least common multiple (LCM) of 18 and 45.
8. Find the least common multiple (LCM) of 42 and 70.
9. Find the least common multiple (LCM) of 120 and 216.
10. Find the least common multiple (LCM) of 24, 42, and 60.
11. Find the least common multiple (LCM) of 84, 108, and 120,
12. Find the least common multiple (LCM) of 504, 756, and 924.
13. Find the greatest common factor (GCF) of 18 and 24.
14. Find the greatest common factor (GCF) of 45 and 63.
15. Find the greatest common factor (GCF) of 168 and 280.
16. Find the greatest common factor (GCF) of 36, 90, and 108.
17. Find the greatest common factor (GCF) of 84, 168, and 252.
18. Find the greatest common factor (GCF) of 162, 270, and 486.
19. For a school lunch program, fruits can be bought in lots of 42, small boxes of milk can be bought in boxes of 30, and sandwiches can be bought in packages of 18. If each student receives one fruit, one box of milk, and one sandwich, what is the least number of students that can be served so that there are no leftover fruits, milk, or sandwiches?

20. Alonso can drive around a race track in 6 minutes and Hamilton takes 8 minutes to drive around the same track. If they both start at the same time, how long will it take until Alonso and Hamilton pass the starting point together?

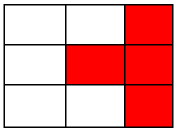
21. Jeff has 72 exams and Jack has 48 exams to grade. Both Jeff and Jack want to put their exams in equal sized groups so that the teaching assistants can grade the exams for them. What is the largest number of exams that they can put in each group so that each group is the same size?

22. Miss Olga has 90 lollipops, 126 candy canes, and 108 chocolate bars that she wants to give away in her classes. If she wants to split each of the candies into same sized piles with none leftover, what is the largest number of lollipops, candy canes, and chocolate bars that she can have in each pile?

Rational numbers are numbers that can be written as a quotient (or fraction) of two integers. Remember, **integers** are whole numbers and their opposites or negatives. Examples of rational numbers include $\frac{1}{2}, \frac{7}{3}, 5, -11, 4\frac{1}{8}, 0, etc.$

The integers are all rational numbers because they can be written as a fraction of integers. For instance, $5 = \frac{5}{1}$ and $-11 = \frac{-11}{1}$.

Fractions are used to represent equal parts of a whole.



There are 4 out of 9 of the equal-sized boxes are shaded. We can represent the shaded part of the whole figure as the fraction $\frac{4}{9}$. Here, the **numerator** is 4, and the **denominator** is 9.

Proper fractions have a value less than one.

Examples of proper fractions are $\frac{2}{3}, \frac{3}{4}, \frac{7}{24}$. The numerator is less than the denominator.

Improper fractions have a value greater than or equal to one.

Examples of improper fractions are $\frac{5}{3}, \frac{7}{7}, \frac{19}{3}$. The numerator is greater than or equal to the denominator.

Mixed numbers have a whole number part and a proper fraction part.

Examples of mixed numbers are $3\frac{1}{2}, 5\frac{3}{4}, 11\frac{7}{23}$. The value of the mixed number is the sum of the whole number and the proper fraction. So, $3\frac{1}{2} = 3 + \frac{1}{2}$, $5\frac{3}{4} = 5 + \frac{3}{4}$, and $11\frac{7}{23} = 11 + \frac{7}{23}$

Writing a mixed number as a improper fraction:

1.New Numerator:

Multiply whole number by the denominator and add the numerator.

2. New denominator:

Keep old denominator.

Example

1. Write $5\frac{2}{3}$ as an improper fraction.

Steps	Reasons
$5\cdot 3+2=17$	Multiply whole number by the denominator and add the numerator.
$\frac{17}{3}$	Keep old denominator.

This process works because $5\frac{2}{3}$ means $5+\frac{2}{3}$. To add the fractions we need a common denominator, which we do as follows $\frac{5}{1}\cdot\frac{3}{3}+\frac{2}{3}$. Looking at how fractions are added shows us why we multiply $5\cdot 3+2$ to get the new numerator and keep the old denominator when changing the mixed number to an improper fraction.

Write an improper fraction as a mixed number

Do long division.

The mixed number is $quotient\,\frac{remainder}{divisor}$.

Example

2. Write $\frac{37}{6}$ as a mixed number.

Steps	Reasons
$\begin{array}{r} 6r1 \\ 6\overline{)37} \\ \underline{-36} \\ 1 \end{array}$	Do long division. Write the remainder of 1 over the 6, which is called the divisor. To get the fractional part of the mixed number
$6\frac{1}{6}$	For the mixed number write $\text{quotient}\frac{\text{remainder}}{\text{divisor}}$

To convert mixed numbers to decimals

Keep the whole number part and divide the fraction to get the decimal part. If the fraction is a repeating decimal, identify the block that is repeating and put a line over it.

Example

3. Convert $5\frac{7}{9}$ to a decimal number.

Steps	Reasons
$5\frac{7}{9} = 5 + \frac{7}{9}$	Split the mixed number into a whole number and the fraction part.
$\begin{array}{r} .777 \\ 9\overline{)7.0000} \\ -\underline{63} \\ 70 \\ -\underline{63} \\ 70 \\ -\underline{63} \\ 7 \end{array}$	Divide the fractional part. When the remainder is the same as in a previous step in the division, the decimal part repeats.
$5.\overline{7}$	Write the whole number part, decimal point, and fractional part as a decimal. Since there is a repeating block for the decimal part, draw a line over it to show that it repeats forever.

To convert decimals to fractions

Decimals can be thought of as fractions with a whole number in the numerator (top) and the place value of the digit farthest to the right for the denominator (bottom).

Example

4. Convert 0.325 to a fraction.

Steps	Reasons
$\frac{325}{1000}$	The last digit is the thousandths. Write the decimal part as a fraction with 1000 in the denominator.
$\frac{5 \cdot 5 \cdot 13}{5 \cdot 5 \cdot 40}$	Factor and cancel the common factors.
$\frac{13}{40}$	Write the simplified fraction.

Equivalent fractions have the same value. $\frac{1}{2}, \frac{3}{6}, \frac{20}{40}, \frac{50}{100}$ are all equivalent fractions. When we multiply a fraction in the numerator and denominator by the same number, we are not changing the value of the fraction.

$\frac{1}{2} = \frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$ $\frac{3}{3}$ is one. Multiplying by one does not change the value.

$\frac{1}{2} = \frac{1}{2} \cdot \frac{20}{20} = \frac{20}{40}$ $\frac{20}{20}$ is one. Multiplying by one does not change the value.

$\frac{1}{2} = \frac{1}{2} \cdot \frac{50}{50} = \frac{50}{100}$ $\frac{50}{50}$ is one. Multiplying by one does not change the value.

To **write in simplest form** we are going in the opposite direction. We cancel the common factor.

Example

5. Write $\frac{20}{25}$ in simplest form.

Steps	Reasons
$\frac{20}{25} = \frac{4 \cdot 5}{5 \cdot 5}$	Factor numerator and denominator using the Greatest Common Factor (GCF).
$\frac{4 \cdot 5}{5 \cdot 5} = \frac{4}{5}$	Cancel the common factor, which is 5. Remember, $\frac{5}{5} = 1$
$\frac{4}{5}$	Write the simplified fraction.

Multiply fractions:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

To multiply fractions, we multiply straight across. The numerators are multiplied to get the new numerator and the denominators are multiplied to get the new denominator. However, instead of just multiplying across it is easier to cancel any common factor in the numerator with any common factor in the denominator in the beginning.

Examples

6. Multiply: $\frac{5}{12}\cdot\frac{24}{25}$ First we show the longer way and then afterwards we will show that the short-cut gives the same answer.

Steps	Reasons
$\frac{5}{12}\cdot\frac{24}{25}$	The longer way is to start by multiplying the two numerators and multiplying the two denominators.
$\frac{5\cdot 24}{12\cdot 25}$	Multiply the numerators and denominators.
$\frac{120}{300}$	Do arithmetic.
$\frac{2\cdot 60}{5\cdot 60}$	Find the Greatest Common Factor (GCF).
$\frac{2}{5}$	When we cancel the common factor we get the answer in reduced form.

There is an easier way to multiply fractions. Instead of multiplying and then cancelling common factors, we can cancel before multiplying, which lets us cancel the numbers while they are smaller.

7. Multiply: $\frac{5}{12}\cdot\frac{24}{25}$

Steps	Reasons
$\frac{5}{12}\cdot\frac{24}{25}$	
$\frac{{}^{1}\cancel{5}\cdot\cancel{24}^{2}}{{}_{1}\cancel{12}\cdot\cancel{25}_{5}}$	24 and 12 have a common factor of 12. So, we cancel the factor in the numerator and denominator. 5 and 25 have a common factor of 5. So, we cancel the 5 in the numerator and denominator.
$\frac{2}{5}$	When we cancel the common factors, we get the answer in reduced form.

8. Find the product of $-\frac{5}{12}, \frac{1}{3}, and -\frac{8}{15}$

Steps	Reasons
$-\frac{5}{12}\left(\frac{1}{3}\right)\left(-\frac{8}{15}\right)$	Product is the result of multiplication. Since two factors are negative the product is positive.
$\frac{5}{3\cdot 4}\cdot\frac{1}{3}\cdot\frac{2\cdot 4}{3\cdot 5}$	Drop all negative signs because the product is positive when there are two negative factors. Factor so that we can cancel common factors.
$\frac{1\cdot 2}{3\cdot 3\cdot 3}$	Cancel a common factor of any numerator and any denominator. The 5´s cancel and the 4´s cancel.
$\frac{2}{27}$	Multiply numerators and multiply denominators.

The easiest way to multiply and simplify is by canceling a common factor as you go.

$$\frac{{}^{1}\cancel{5}}{{}_{3}\cancel{12}}\left(\frac{1}{3}\right)\left(\frac{\cancel{8}^{2}}{\cancel{15}_{3}}\right)=\frac{2}{27}$$

8 and 12 have a common factor of 4. 5 and 15 have a common factor of 5.

Dividing fractions:

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c} \quad \text{or} \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\cdot\frac{d}{c}$$

To divide fractions we take the reciprocal of the divisor and multiply. That is to say we flip the fraction after the ÷ symbol and multiply (or flip the fraction in the denominator).

Examples

9. Divide $\frac{3}{8}\div\frac{9}{16}$

Steps	Reasons
$\frac{3}{8}\cdot\frac{16}{9}$	Multiply by the reciprocal of the divisor. In other words, flip the second fraction and multiply.
$\frac{3}{8}\cdot\frac{8\cdot 2}{3\cdot 3}$	Factor to cancel common factors.
$\frac{2}{3}$	3´s and 8´s cancel.

10. Find the quotient of $-3\frac{5}{11}$ and $3\frac{4}{5}$.

Steps	Reasons
$-3\frac{5}{11} \div 3\frac{4}{5}$	Quotient indicates that we are dividing.
$-\frac{38}{11} \div \frac{19}{5}$	Write the mixed numbers as improper fractions. A negative number divided by a positive number is negative.
$-\frac{38}{11} \cdot \frac{5}{19}$	Multiply by the reciprocal of the divisor. In other words, flip the second fraction and multiply.
$-\frac{2 \cdot 19}{11} \cdot \frac{5}{19}$	Factor in order to cancel the common factor of 19.
$-\frac{2}{11} \cdot \frac{5}{1}$	Cancel the common factor 19.
$-\frac{10}{11}$	The answer is a reduced proper fraction. If the answer is an improper fraction, we need to do division in order to write the answer as a mixed number. Keep the negative sign!

Deciding the order of fractions (< or >) :

1. Write equivalent fractions with the same denominator. We are looking for the least common multiple (LCM) of the denominators.
2. Compare the numerators.

Example

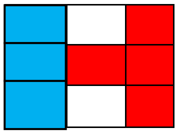
11. Place the correct symbol < or > between $\frac{7}{8} \quad \frac{13}{16}$.

Steps	Reasons
$\frac{7}{8} = \frac{7}{8} \cdot \frac{2}{2} = \frac{14}{16}$	16 is the Least Common Multiple (LCM) of the denominators. So, we write $\frac{7}{8}$ as an equivalent fraction with denominator of 16
$\frac{14}{16} > \frac{13}{16}$	The larger numerator is the larger number if the denominators are the same.
$\frac{7}{8} > \frac{13}{16}$	Write the original fractions with the correct order.

Adding fractions:

1. Find the least common denominator (or LCD) of the denominators.
2. Get the LCD for every fraction by multiplying the same numbers in the numerator (top) and denominator (bottom). That way the fractions are equivalent.
3. Add numerators and keep the common denominator.

If we add $\frac{4}{9}$ and $\frac{3}{9}$, we get $\frac{7}{9}$ as in the picture of the shaded squares:



There are seven out of nine shaded regions (or $\frac{7}{9}$) when we add the four shaded regions to the three shaded regions.

Examples

12. Add $\frac{3}{8}+\frac{1}{6}$

Steps	Reasons
$\frac{3}{8}+\frac{1}{6}$	1. Find the LCD of 8 and 6, which is 24 because 24 is the least common multiple of 8 and 6.
$\frac{3}{8}\cdot\frac{3}{3}+\frac{1}{6}\cdot\frac{4}{4}$	2. Get the LCD by multiplying the same numbers in the numerator (top) and denominator (bottom).
$\frac{9}{24}+\frac{4}{24}$	3. Add numerators and keep the common denominator.
$\frac{13}{24}$	

13. Find the sum of $-\frac{3}{4}, \frac{5}{6}, and\ \frac{2}{3}$

Steps	Reasons
$-\frac{3}{4}+\frac{5}{6}+\frac{2}{3}$	Sum means we add.
$-\frac{3}{4}\left(\frac{3}{3}\right)+\frac{5}{6}\left(\frac{2}{2}\right)+\frac{2}{3}\left(\frac{4}{4}\right)$	We need to get the Least Common Denominator (LCD), which is 12. Multiply the numerator and denominator by the same number.
$-\frac{9}{12}+\frac{10}{12}+\frac{8}{12}$	Do the multiplication in the numerator and denominator.
$\frac{-9+10+8}{12}$	Add the numerators and keep the LCD
$\frac{\cancel{9}^{3}}{\cancel{12}_{4}}$	Do the addition and simplify the fraction by cancelling the common factor of 3.
$\frac{3}{4}$	Write the answer.

14. Subtract $-\frac{3}{10}-\left(-\frac{5}{6}\right)$

Steps	Reasons
$-\frac{3}{10}+\frac{5}{6}$	Change minus a negative to addition.
$-\frac{3}{10}\left(\frac{3}{3}\right)+\frac{5}{6}\left(\frac{5}{5}\right)$	Get the LCD. By multiplying the same number in the numerator and denominator.
$-\frac{9}{30}+\frac{25}{30}$	Multiply in the numerator and the denominator
$\frac{-9+25}{30}$	Add the numerators. Keep the negative sign in the numerator. Keep the LCD
$\frac{16}{30}$	Do the addition.
$\frac{8\cdot 2}{15\cdot 2}$ or $\frac{8}{15}$	Simplify by factoring out the greatest common factor of 2 and cancel the common factor to get the answer.

15. Add $7\tfrac{1}{5}-4\tfrac{2}{3}$

Steps	Reasons
$7\tfrac{1}{5}-4\tfrac{2}{3}$	
$\frac{36}{5}-\frac{14}{3}$	Convert the mixed numbers to improper fractions: $7\cdot 5+1=36$ and $4\cdot 3+2$
$\frac{36}{5}\left(\frac{3}{3}\right)-\frac{14}{3}\left(\frac{5}{5}\right)$	Get the LCD. By multiplying the same number in the numerator and denominator.
$\frac{108}{15}-\frac{70}{15}$	Multiply in the numerator and the denominator.
$\frac{38}{15}$	Subtract the numerators and keep the LCD.
$15\overline{)38}$ quotient $2\tfrac{8}{15}$ $-\underline{30}$ 8	Divide to change the improper fraction to a mixed number.
$2\tfrac{8}{15}$	Write the answer.

For exponents and fractions just write out the multiplication.

Examples

16. Evaluate $\left(\frac{3}{4}\right)^3$

Steps	Reasons
$\left(\frac{3}{4}\right)^3$	$\frac{3}{4}$ is multiplied by itself three times.
$\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)$	Write out the multiplication.
$\frac{27}{64}$	Multiply the fractions by multiply the numerators and the denominators. There is no common factor to cancel.

17. Evaluate $\left(-2\frac{1}{5}\right)^2$

Steps	Reasons
$\left(-2\frac{1}{5}\right)^2$	The $-2\frac{1}{5}$ is multiplied by itself twice
$\left(-2\frac{1}{5}\right)\left(-2\frac{1}{5}\right)$	Write out the multiplication.
$\left(\frac{11}{5}\right)\left(\frac{11}{5}\right)$	Write the mixed numbers as improper fractions. The product of two negative numbers is positive. So, we can drop the negative signs.
$\frac{121}{25}$	Multiply the fractions.
$4\frac{21}{25}$	Rewrite as a mixed number by dividing.

Things can get a little tricky when we have complex fractions or lots of operations to perform. The key is to do the same operations in the same order as one would with whole numbers. Since the arithmetic operations with fractions are more complicated, you may need to work out the fractions on the side. For instance, 2+3 can be done in your head and poses no problems in the middle of a more complicated problem. However, $2\frac{1}{3}+3\frac{4}{5}$ is difficult to do mentally since it requires several steps.

When doing the next few examples it is important to keep in mind the order of operations.

The **order of operations** is as follows:

1. **Parentheses** inside to outside.
2. **Exponents.**
3. **Multiplication** and **division** together as they appear from left to right.
4. **Addition** and **subtraction** together as they appear from left to right.

Some people remember **PEMDAS** to help keep the order straight.

Examples

18. Simplify $\dfrac{\frac{2}{3}-\frac{3}{4}}{\frac{1}{2}+\frac{3}{5}}$

<u>Steps</u> <u>Reasons</u>

$$\dfrac{\frac{2}{3}-\frac{3}{4}}{\frac{1}{2}+\frac{3}{5}}$$

The key to this problem is to treat it like three problems. First subtract in the numerator. Second add in the denominator. Third do division.

$$\dfrac{\left(\frac{2}{3}-\frac{3}{4}\right)}{\left(\frac{1}{2}+\frac{3}{5}\right)}$$

The numerator is one part of the fraction and the denominator is another part. So, we can place parentheses around the numerator and denominator, which helps us see the order of operations. Normally, we would not take this step to write the parentheses.

First do the subtraction in the numerator. Second add in the denominator. You may do these two steps neatly to the side or at the same time that you do the subtraction in the numerator.

$$\dfrac{-\frac{1}{12}}{\frac{11}{10}}$$

$$\frac{2}{3}-\frac{3}{4}$$
$$\frac{2}{3}\cdot\frac{4}{4}-\frac{3}{4}\cdot\frac{3}{3}$$
$$\frac{8}{12}-\frac{9}{12}$$
$$-\frac{1}{12}$$

and

$$\frac{1}{2}+\frac{3}{5}$$
$$\frac{1}{2}\cdot\frac{5}{5}+\frac{3}{5}\cdot\frac{2}{2}$$
$$\frac{5}{10}+\frac{6}{10}$$
$$\frac{11}{10}$$

Put the fractions in the numerator and denominator. Now, there is a fraction divided by a fraction.

$$-\frac{1}{12}\cdot\frac{10}{11}$$

Multiply by the reciprocal of the denominator. (Flip the bottom fraction and multiply.)

$$-\frac{1\cdot\cancel{2}\cdot 5}{\cancel{2}\cdot 6\cdot 11}$$

Factor and cancel the common factor.

$$-\frac{5}{66}$$

Multiply in the numerator and denominator.

19. Simplify $\frac{2}{3}+\frac{\frac{5}{36}}{5-\frac{1}{5}}\div\frac{5}{36}$

Steps	Reasons
$\frac{2}{3}+\frac{\frac{5}{36}}{5-\frac{1}{5}}\div\frac{5}{36}$	The key is to break this problem down into smaller problems. The first step is to simplify the complex fraction.
	Do subtraction in the denominator on the side. Just ignore $\frac{5}{36}$ until the subtraction is complete. $5-\frac{1}{5}$
$\frac{2}{3}+\frac{\frac{5}{36}}{\frac{24}{5}}\div\frac{5}{36}$	$\frac{25}{5}-\frac{1}{5}$ $\frac{24}{5}$ When the subtraction in the denominator is finished put the $\frac{24}{5}$ back into the complex fraction.
$\frac{2}{3}+\frac{25}{864}\div\frac{5}{36}$	Do division: $\frac{\frac{5}{36}}{\frac{24}{5}}=\frac{5}{36}\cdot\frac{5}{24}=\frac{25}{864}$ Put the fraction back into the original problem.
$\frac{2}{3}+\frac{25}{864}\cdot\frac{36}{5}$	Rewrite the division as multiplication.
$\frac{2}{3}+\frac{25}{864}\cdot\frac{36}{5}$	Cancel the common factors of 36 and 5.
$\frac{2}{3}\cdot\frac{8}{8}+\frac{5}{24}$	Add fractions by getting the LCD.
$\frac{16}{24}+\frac{5}{24}=\frac{21}{24}\ or\ \frac{7}{8}$	Add numerators, keep the LCD, and then simplify if possible.

20. Evaluate $3\cdot\left(\frac{5}{6}\right)^2\cdot\left(-\frac{2}{5}\right)^3$

Steps	Reasons
$\frac{3}{1}\cdot\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\cdot\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right)$	Write the whole number as an improper fraction. Rewrite the fractions without exponents.
$-\frac{\cancel{3}\cdot\cancel{5}\cdot\cancel{5}\cdot\cancel{2}\cdot\cancel{2}\cdot 2}{1\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{2}\cdot 3\cdot\cancel{5}\cdot\cancel{5}\cdot 5}$	Three factors are negative. So, the product is negative. Cancel the common factors.
$-\frac{2}{15}$	The two is left in the numerator. Multiply the remaining factors in the denominator.

Exercises:

Write as an improper fraction:

1. $3\frac{1}{2}$

2. $2\frac{3}{5}$

3. $6\frac{5}{8}$

4. $12\frac{3}{4}$

5. $-5\frac{4}{7}$

6. $-4\frac{5}{8}$

7. $-15\frac{2}{7}$

Write as a mixed number:

8. $\frac{7}{3}$

9. $\frac{23}{5}$

10. $\frac{112}{6}$

11. $\frac{77}{9}$

12. $\frac{-38}{7}$

13. $-\frac{57}{12}$

14. $-\frac{128}{15}$

Convert to a decimal number:

15. $\frac{5}{8}$

16. $\frac{9}{12}$

17. $\frac{28}{3}$

18. $-\frac{7}{11}$

19. $3\frac{1}{5}$

20. $8\frac{7}{25}$

21. $-12\frac{3}{16}$

22. $-14\frac{15}{24}$

23. $-16\frac{5}{9}$

24. $7\frac{11}{27}$

25. $32\frac{5}{6}$

26. $-15\frac{7}{11}$

Convert to a fraction:

27. 0.57

28. 0.725

29. 5.32

30. 4.75

31. 0.562

32. 0.375

Write in simplest form:

33. $\frac{12}{28}$

34. $\frac{42}{48}$

35. $\frac{125}{150}$

36. $\frac{121}{165}$

37. $\frac{84}{168}$

38. $\frac{54}{90}$

Simplify by multiplying, dividing, adding, subtracting and using the order of operations when appropriate:

39. $\frac{3}{5} \cdot \frac{4}{9}$

40. $\frac{5}{12} \cdot \frac{9}{25}$

41. $\frac{7}{27} \cdot \frac{18}{42}$

42. $-\frac{8}{27} \cdot \frac{3}{16}$

43. $\frac{30}{56} \cdot \left(-\frac{16}{45}\right)$

44. $\left(-\frac{42}{63}\right) \cdot \left(-\frac{45}{70}\right)$

45. $\left(-\frac{5}{12}\right) \cdot \left(-\frac{8}{25}\right)$

46. $3\frac{2}{3} \cdot 5\frac{1}{4}$

47. $4\frac{3}{8} \cdot 2\frac{4}{5}$

48. $\left(-1\frac{5}{8}\right)\cdot\left(-\ 3\frac{5}{7}\right)$

49. $5\frac{1}{9}\cdot\left(-7\frac{2}{3}\right)$

50. $-10\frac{3}{4}\cdot 2\frac{8}{9}$

51. $\left(-6\frac{2}{7}\right)\left(-4\frac{3}{8}\right)$

52. $\frac{12}{25}\div\frac{6}{15}$

53. $\frac{9}{14}\div\frac{27}{35}$

54. $\frac{22}{42}\div\frac{33}{54}$

55. $-\frac{8}{27}\div\frac{16}{81}$

56. $\frac{25}{54}\div\left(-\frac{10}{36}\right)$

57. $-\frac{16}{25}\div\left(-\frac{15}{8}\right)$

58. $\frac{-\frac{4}{5}}{\frac{10}{11}}$

59. $\frac{\frac{4}{7}}{-\frac{12}{28}}$

60. $4\frac{2}{3}\div 2\frac{2}{5}$

61. $5\frac{5}{8}\div 3\frac{1}{3}$

62. $7\frac{2}{9}\div\left(-2\frac{6}{7}\right)$

63. $-12\frac{1}{2}\div 6\frac{2}{3}$

64. $-5\frac{1}{3}\div 2\frac{2}{7}$

65. $-3\frac{5}{6} \div \left(-11\frac{1}{2}\right)$

66. $-3\frac{7}{12} \div \left(-10\frac{6}{8}\right)$

67. $\left(\frac{2}{3}\right)^2$

68. $\left(\frac{3}{5}\right)^3$

69. $\left(-\frac{5}{7}\right)^2$

70. $\left(3\frac{1}{3}\right)^2$

71. $\left(-2\frac{3}{4}\right)^2$

72. $\left(-1\frac{3}{5}\right)^3$

73. $\frac{3}{7} + \frac{2}{7}$

74. $\frac{1}{5} + \frac{3}{5}$

75. $\frac{7}{9} - \frac{2}{9}$

76. $\frac{7}{11} - \frac{3}{11}$

77. $\frac{1}{4} + \frac{2}{3}$

78. $\frac{5}{6} + \frac{1}{12}$

79. $\frac{7}{12} - \frac{4}{15}$

80. $\frac{5}{18} - \frac{19}{30}$

81. $-\frac{5}{6} - \left(-\frac{3}{10}\right)$

82. $-\frac{5}{12}-\left(-\frac{7}{18}\right)$

83. $-\frac{11}{15}-\left(\frac{4}{25}\right)$

84. $3\frac{1}{4}-1\frac{1}{3}$

85. $2\frac{5}{6}+11\ \frac{2}{3}$

86. $7\frac{1}{3}+2\frac{3}{5}$

87. $4\frac{2}{9}-8\frac{5}{6}$

88. $9\frac{5}{12}-12\frac{11}{18}$

89. $3\frac{5}{6}+4\frac{3}{4}$

90. $5\frac{2}{5}+3\frac{7}{10}$

91. $3\frac{1}{6}-5\frac{2}{9}$

92. $\frac{\frac{2}{3}+\frac{1}{4}}{\frac{3}{4}-\frac{1}{6}}$

93. $\frac{\frac{5}{14}-\frac{4}{7}}{\frac{4}{21}+\frac{11}{3}}$

94. $\frac{\frac{3}{5}+\frac{1}{2}}{\frac{2}{3}-\frac{5}{6}}$

95. $\frac{\frac{1}{2}-\frac{2}{3}}{\frac{3}{5}+\frac{1}{6}}$

96. $\frac{3}{5}+\frac{4-\frac{2}{3}}{\frac{25}{9}}\div\frac{7}{2}$

97. $\frac{1}{2}+\frac{\frac{3}{8}}{\frac{5}{2}-4}\cdot\frac{2}{3}$

98. $\frac{\frac{5}{6}}{\frac{2}{3}-2}+\frac{1}{6}\cdot\frac{3}{4}$

99. $\frac{\frac{19}{5}}{3+\frac{1}{6}}-\frac{7}{8}\div\frac{3}{2}$

100. $\frac{4}{9}-\frac{3-\frac{3}{4}}{\frac{3}{4}}\div 6$

Find the answer:

101. Find the product of $-\frac{5}{6}$, $\frac{7}{15}$, and $\frac{9}{10}$.

102. Find the quotient of $5\frac{3}{4}$ and $-3\frac{1}{5}$.

103. Find the sum of $\frac{2}{3}$, $\frac{4}{9}$, and $-\frac{5}{6}$.

104. Find the difference of $\frac{7}{12}$ and $\frac{4}{15}$.

105. Find the quotient of $\frac{11}{27}$ and $\frac{5}{18}$.

106. Find the sum of $3\frac{2}{3}$, $-5\frac{7}{9}$, and $-3\frac{5}{6}$.

107. Find the difference of $-4\frac{4}{7}$ and $2\frac{3}{14}$.

108. Find the product of $-2\frac{1}{3}$, $5\frac{2}{5}$, and $-3\frac{1}{2}$.

109. If it takes $12\frac{2}{3}$ gallons of gas to fill a car's tank, how much gas does it take to fill $\frac{3}{5}$ of the tank with gas?

110. If it takes $15\frac{3}{4}$ gallons of gas to fill a car's tank, how much gas does it take to fill $\frac{2}{3}$ of the tank with gas?

If we square a whole number, we get what is called a perfect square:

Whole Number	Perfect Square	Whole Number Squared
0	0	0^2
1	1	1^2
2	4	2^2
3	9	3^2
4	16	4^2
5	25	5^2
6	36	6^2
7	49	7^2
8	64	8^2
9	81	9^2
10	100	10^2
11	121	11^2
12	144	12^2

If we know our multiplication table, we should be able to remember this table as well. Squaring is going from 7 to 49 because $7^2 = 49$. Taking the square root is going in the opposite direction from 49 to 7.

The square root of a number a is the number whose square is a.
Symbolically, c is a square root of a if $c^2 = a$

For example, the square roots of 9 are 3 and –3 because $3^2 = 9$ and $(-3)^2 = 9$.

The symbol $\sqrt{\ }$ refers to the positive square root. So, $\sqrt{9} = 3$. We generally will not worry about the negative square root.

Examples

1. Find $\sqrt{25}$

$\sqrt{25} = 5$ because $5^2 = 25$

2. Find $\sqrt{64}$

$\sqrt{64} = 8$ because $8^2 = 64$

3. Find $\sqrt{20}$

$\sqrt{20}$ is not a perfect square. So, there is no whole number square root.
We can use a calculator to get an approximation 4.4721…

For a whole number the square root is either:
1. another whole number like $\sqrt{25} = 5$ and $\sqrt{64} = 8$
2. an irrational number

An irrational number cannot be written as a fraction of integers. If we try to write an irrational number as a decimal number, it goes on forever without repeating, which is impossible to write. A classic example of an irrational number is π, which goes on forever without repeating. π is about 3.14159…, but if we want to indicate its exact value we need to use the symbol π. Many square roots are similar, but we can simplify many of them.

Product Rule for square roots:

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

To simplify square roots factor out a factor that has a perfect square.

Examples

4. Simplify $\sqrt{20}$

Steps	Reasons
$\sqrt{20}$	Look for a factor of 20 with a square root. We can guess and check from the list of perfect squares above. Take the square root of
$\sqrt{4}\sqrt{5}$	
$2\sqrt{5}$	
$2\sqrt{5}$	Here we have the “simplified” form because we have pulled out as much as we can from underneath the square root.

5. Simplify: $\sqrt{180}$

$\sqrt{180} = \sqrt{} \cdot \sqrt{}$ The idea is to factor 180 so that one factor has a root that is a whole number and the other factor does not.

5
$3\overline{)15}$
$3\overline{)45}$
$2\overline{)90}$
$2\overline{)180}$

Most of us do not see that 36 is the largest factor of 180 that has a whole number root. Try the following:
1. Find the prime factorization
2. Group the pairs of factors in the prime factorization. $(2^2 \cdot 3^2)(5)$
3. These pairs of prime factors will have a square root. $(2^2 \cdot 3^2)(5) = (36)(5)$

$\sqrt{180} = \sqrt{36} \cdot \sqrt{5}$ 36 has a square root and 5 is left-over

$6\sqrt{5}$ This is the simplified form. We have pulled out as much as we can from under the square root.

6. Simplify: $5\sqrt{162}$

$\sqrt{162} = \sqrt{} \cdot \sqrt{}$ The idea is to factor 162 so that one factor has a root that is a whole number and the other factor does not.

3
$3\overline{)9}$
$3\overline{)27}$
$3\overline{)81}$
$2\overline{)162}$

Most of us do not see that 81 is the largest factor of 162 that has a whole number root. Try the following:
1. Find the prime factorization
2. Group the pairs of factors in the prime factorization. $(3^4)(2)$
3. These pairs of prime factors will have a square root. $(3^4)(2) = (81)(2)$

$5\sqrt{81} \cdot \sqrt{2}$ 81 has a square root and 2 is left-over

$5 \cdot 9\sqrt{2}$ We have pulled out as much as we can from under the square root.

$45\sqrt{2}$ Multiply the numbers.

<u>Multiplying roots</u> by using the same rule as we used to simplify: $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$

Examples

7. Multiply $\sqrt{15} \cdot \sqrt{30}$

Steps	Reasons
$\sqrt{15} \cdot \sqrt{30}$	First multiply the parts under the square root because of the rule $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$
$\sqrt{450}$	If you do not see that 225 is a perfect square factor of 450, finding the prime factorization may help.
$\sqrt{225} \cdot \sqrt{2}$	$450 = 2 \cdot 3^2 \cdot 5^2$ So, $3^2 \cdot 5^2 = 225$ will have a square root. Also guessing the part of the expression that does not have a square root (2,3,5,6,7,8,…) and using your calculator will often work.
$15\sqrt{2}$	

A good trick is to multiply numbers under the root (inside the house) and multiply the numbers outside of the root (outside of the house).

8. Multiply $3\sqrt{6} \cdot 5\sqrt{10}$

Steps	Reasons
$3\sqrt{6} \cdot 5\sqrt{10} = 15\sqrt{60}$	The original problem is 3 times $\sqrt{6}$ times 5 times $\sqrt{10}$. Because the only operation is multiplication we can multiply in any order (commutative property).
$= 15\sqrt{4}\sqrt{15}$	Multiply under the square root (6·10=60) and multiply outside of the square root (3·5=15).
$= 15 \cdot 2 \cdot \sqrt{15}$	Simplify square root of 60 because 4 is a perfect square factor of 60.
$= 30 \cdot \sqrt{15}$	

Distributive Property:

$a(b + c) = a \cdot b + a \cdot c$

We can use the distributive property when multiplying roots.

To add and subtract square root expressions we need to think of like terms. The root must be exactly the same. Then add or subtract the numbers in front (coefficients) and keep the same root part. It works as follows:

$5\sqrt{3}+2\sqrt{3}=7\sqrt{3}$ $5x + 2x = 7x$	Add the numbers and keep the same root part just like we do with variables.

For $3\sqrt{2}+8\sqrt{3}$ We cannot simplify further because the numbers under the square root (radicands) are different.

Examples

9. Multiply: $3\sqrt{5}\left(2\sqrt{5}+\sqrt{2}\right)$

Steps	Reasons
$3\sqrt{5}\left(2\sqrt{5}+\sqrt{2}\right)=3\sqrt{5}\cdot 2\sqrt{5}+3\sqrt{5}\cdot\sqrt{2}$	We cannot add inside the parentheses. Use the distributive property to multiply both terms inside the parentheses by the $3\sqrt{5}$, which is outside of the parentheses.
$=6\sqrt{25}+3\sqrt{10}$	
$=6\cdot 5+3\sqrt{10}$	Multiply under the square root (5·5=25 and 5·2=10). Multiply outside of the square root (3·2=6 and 3·1=3). Simplify the first root.
$=30+3\sqrt{10}$	

10. Simplify: $3\sqrt{50}-2\sqrt{18}+4\sqrt{98}$

Steps	Reasons
$3\sqrt{50}-2\sqrt{18}+4\sqrt{98}$	Since there are different numbers under the square root (radicands), we cannot collect like terms right away.
$3\sqrt{25}\sqrt{2}-2\sqrt{9}\sqrt{2}+4\sqrt{49}\sqrt{2}$	1. Simplify each root by finding a factor that is a perfect square.
$3{\cdot}5\sqrt{2}-2{\cdot}3\sqrt{2}+4{\cdot}7\sqrt{2}$	2. Take the square root each perfect square.
$15\sqrt{2}-6\sqrt{2}+28\sqrt{2}$	3. Multiply.
$37\sqrt{2}$	4. Now the roots are exactly the same. So, add and subtract the numbers and keep the $\sqrt{2}$

Exercises

Simplify by finding the square root. If the square root is not a whole number, use a calculator and round to the thousandth.

1. $\sqrt{49}$

2. $\sqrt{81}$

3. $\sqrt{64}$

4. $5\sqrt{121}$

5. $9\sqrt{16}$

6. $\sqrt{731}$

7. $\sqrt{157}$

8. $3\sqrt{12{,}895}$

9. $21\sqrt{45{,}693}$

Simplify by finding a factor that is a perfect square.

10. $\sqrt{75}$

11. $\sqrt{32}$

12. $\sqrt{108}$

13. $\sqrt{128}$

14. $8\sqrt{28}$

15. $5\sqrt{27}$

16. $3\sqrt{63}$

17. $15\sqrt{242}$

18. $21\sqrt{147}$

19. $\sqrt{15} \cdot \sqrt{6}$

20. $\sqrt{35} \cdot \sqrt{10}$

21. $\sqrt{30} \cdot \sqrt{70}$

22. $\sqrt{21} \cdot \sqrt{7}$

23. $7\sqrt{6} \cdot 7\sqrt{8}$

24. $4\sqrt{8} \cdot 3\sqrt{14}$

25. $7\sqrt{24} \cdot 5\sqrt{36}$

26. $5\sqrt{10} \cdot 6\sqrt{20}$

27. $\sqrt{5} \cdot \left(\sqrt{15} + \sqrt{5}\right)$

28. $\sqrt{3} \cdot \left(\sqrt{27} + \sqrt{6}\right)$

29. $\sqrt{7} \cdot \left(\sqrt{14} - \sqrt{7}\right)$

30. $\sqrt{6} \cdot \left(\sqrt{10} - \sqrt{2}\right)$

31. $3\sqrt{5} \cdot \left(2\sqrt{5} + 3\sqrt{15}\right)$

32. $4\sqrt{7} \cdot \left(2\sqrt{21} - 6\sqrt{7}\right)$

33. $7\sqrt{2} \cdot \left(5\sqrt{6} - 4\sqrt{2}\right)$

34. $6\sqrt{10} \cdot \left(3\sqrt{15} + 4\sqrt{6}\right)$

35. $4\sqrt{3} + 7\sqrt{3} - 2\sqrt{3}$

36. $8\sqrt{7} - 2\sqrt{7} + 4\sqrt{7}$

37. $3\sqrt{5}+6\sqrt{5}-7\sqrt{10}$

38. $9\sqrt{6}-11\sqrt{2}+15\sqrt{6}$

39. $12\sqrt{12}+3\sqrt{27}-4\sqrt{75}$

40. $3\sqrt{45}-12\sqrt{125}+2\sqrt{80}$

41. $7\sqrt{32}+8\sqrt{50}-10\sqrt{72}$

42. $15\sqrt{108}-12\sqrt{147}+2\sqrt{75}$

For the next two problems, the velocity (v) of a Tsunami in kilometers per hour can be modeled by the formula $\mathrm{v}=100+9.8\sqrt{\mathrm{D}}$ where (D) is the depth of the water measured in meters.

43. a. If the depth is 5000 meters, find the velocity of the tsunami in kilometers per hour to the nearest hundredth.
    b. If the depth is 600 meters, find the velocity of the tsunami to the nearest hundredth of kilometers per hour.
    c. What is the effect on the velocity of the Tsunami as the depth decreases?

44. a. If the depth is 3000 meters, find the velocity of the tsunami in kilometers per hour to the nearest hundredth.
    b. If the depth is 200 meters, find the velocity of the tsunami to the nearest hundredth of kilometers per hour.
    c. What is the effect on the velocity of the Tsunami as the depth decreases?

For the next two problems ignoring air resistance, the time (t) in seconds that it takes a dropped object to fall can be modeled by $\mathrm{t}=\sqrt{0.204\cdot\mathrm{d}}$ where the distance (d) is measured in meters.

45. From the top of the Empire state building to the ground below it is 381 meters. Ignoring air resistance, find the time that it takes for an object to fall 381 meters to the nearest tenth of a second.

46. From the top of the antenna of the Burj Khalifa in Dubai to the ground below it is 828 meters. Ignoring air resistance, find the time that it takes for an object to fall 828 meters to the nearest tenth of a second.

Exponents can be used to write repeated multiplication. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ The exponent of 4 tells us how many times to multiply the base, which is the 3. There are several rules for exponents, which help us to manipulate complicated exponential expressions. After each rule, there is a quick example to help show why the rule is true.

Rules for Exponents:

1. $x^m \cdot x^n = x^{m+n}$

Example: $3^2 \cdot 3^4 = (3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) = 3^6$. It is easier to add the 2 and 4 to get the 6.

2. $(x^m)^n = x^{m \cdot n}$

Example: $(5^3)^2 = (5^3)(5^3) = (5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5) = 5^6$. It is easier to multiply the exponent outside the parentheses by the exponent inside the parentheses.

3. $(x^m y^n)^p = x^{m \cdot p} y^{n \cdot p}$

Example: $(7^3 \cdot 9^4)^2 = (7^3 \cdot 9^4)(7^3 \cdot 9^4) = (7 \cdot 7 \cdot 7 \cdot 9 \cdot 9 \cdot 9 \cdot 9)(7 \cdot 7 \cdot 7 \cdot 9 \cdot 9 \cdot 9 \cdot 9) = 7^6 \cdot 9^8$. It is easier to multiply the exponent outside the parentheses by the exponents inside the parentheses: $3 \cdot 2 = 6$ is the exponent for 7 and $4 \cdot 2 = 8$ is the exponent for 9.

4. $\frac{x^m}{x^n} = x^{m-n}$

Example $\frac{3^6}{3^2} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3} = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$. It is easier to subtract 6 – 2 to get 4.

5. $x^0 = 1$

Example:

$1 = \frac{10^5}{10^5}$	Any number divided by itself is one. (Except for zero.)
$= 10^{5-5}$	By the rule above
$= 10^0$	Subtract 5 – 5 = 0

So, $10^0 = 1$ which works for any base except 0.

6. $x^{-n} = \dfrac{1}{x^n}$

Example:

$\frac{1}{5^3} = \frac{5^0}{5^3}$	Rule 2 says $x^0 = 1$. So, $5^0 = 1$
$= 5^{0-3}$	Rule 1.
$=5^{-3}$	Subtract 0 – 3 = –3

It should be noted that there are some restrictions here. The x and y are real numbers. There can be no zeros in the denominator.

Examples

1. Simplify by using the rules for exponents: $2^3 \cdot 2^5$

Steps	Reasons
$2^3 \cdot 2^5$	Use the rule $x^m \cdot x^n = x^{m+n}$
2^{3+5}	
2^8	Add the exponents because of multiplication with the same base.
256	

2. Simplify by using the rules for exponents: $\dfrac{5^9}{5^6}$

Steps	Reasons
$\frac{5^9}{5^6}$	Use the rule $\frac{x^m}{x^n} = x^{m-n}$
5^{9-6}	
5^3	Subtract the exponents because of division with the same base.
125	

3. Simplify by using the rules for exponents: $\left(3^2\right)^2$

Steps	Reasons
$\left(3^2\right)^2$	Use the rule $\left(x^m\right)^n = x^{m \cdot n}$
$3^{2 \cdot 2}$	
3^4	Multiply the exponents because of exponent outside of the parentheses.
81	

4. Simplify by using the rules for exponents: 5^{-2}

Steps	Reasons
5^{-2}	Negative exponent.
$\frac{1}{5^2}$	Just the rule 6: $x^{-n} = \frac{1}{x^n}$
$\frac{1}{25}$	Five squared is twenty-five.

5. Simplify by using the rules for exponents: -7^{-5}

Steps	Reasons
-7^{-5}	Remember that the negative sign in front is not part of the exponent. It is kept in front and is not affected by the exponent.
$-\frac{1}{7^5}$	For negative exponents use $x^{-n} = \frac{1}{x^n}$

6. Simplify by using the rules for exponents: -20^0

Steps	Reasons
-20^0	Remember that the negative sign in front is not part of the exponent. It is kept in front and is not affected by the exponent.
-1	For exponent of zero $x^0 = 1$

7. Simplify by using the rules for exponents: $\frac{4^9}{4^{11}}$

Steps	Reasons
$\frac{4^9}{4^{11}}$	First use the rule $\frac{x^m}{x^n} = x^{m-n}$
4^{9-11}	Subtract the exponents.
4^{-2}	
$\frac{1}{4^2}$	For a negative exponent, just use $x^{-n} = \frac{1}{x^n}$
$\frac{1}{16}$	Four squared is 16 in the denominator.

8. Simplify by using the rules for exponents: $\frac{x^8y^7}{x^3y^5}$

Steps	Reasons
$\frac{x^8y^7}{x^3y^5}=x^{8-3}y^{7-5}$	Subtract the exponents in the denominator (bottom) from the exponents in the numerator (top). Rule number 4.
$=x^5y^2$	Do the subtraction.

9. Simplify by using the rules for exponents: $\left(\frac{8x^{-5}y^7}{6x^4y^{-6}}\right)^{-2}$

Steps	Reasons
$\left(\frac{\overset{4}{\cancel{8}}x^{-5-4}y^{7+6}}{\underset{3}{\cancel{6}}}\right)^{-2}$	Subtract the exponent in the denominator (bottom) from the exponent in the numerator (top). $\frac{x^m}{x^n}=x^{m-n}$. I think of "cross the line change the sign". That way for y I can write an exponent of 7 + 6 instead of 7 – (–6). Cancel the common factor for the numbers.
$\left(\frac{4x^{-9}y^{13}}{3}\right)^{-2}$	Add and subtract for the exponents.
$\frac{4^{-2}x^{(-2)(-9)}y^{(-2)(13)}}{3^{-2}}$	Multiply the exponent outside the parentheses by all the exponents inside parentheses.
$\frac{3^2x^{18}y^{-26}}{4^2}$	Get rid of the negative exponents by "cross the line change the sign".
$\frac{9x^{18}}{16y^{26}}$	Evaluate the numbers.

10. Simplify by using the rules for exponents: $\frac{(3x^{-2}y^{7})^{-2}}{(9x^{5}y^{-6})^{-1}}$

Steps	Reasons
$\frac{(9x^{5}y^{-6})^{1}}{(3x^{-2}y^{7})^{2}}$	Flipping the numerator and denominator gets rid of the negative exponents outside of the parentheses.
$\frac{9x^{5}y^{-6}}{3^{2}x^{(-2)(2)}y^{7\cdot 2}}$	Multiply the exponent outside of the parentheses by the exponents inside parentheses.
$\frac{\cancel{9}x^{5}y^{-6}}{\cancel{9}x^{-4}y^{14}}$	Do the arithmetic in the denominator.
$x^{5+4}y^{-6-14}$	Cancel the common factor of 9. Bring the variables in the denominator up by "cross the line change the sign."
$x^{9}y^{-20}$ *or* $\frac{x^{9}}{y^{20}}$	Write your answer either with no variables in the denominator or no negative exponents.

Scientific notation

Large and small decimal numbers can be more conveniently written using scientific notation.

$a\times 10^{n}$ a has a digit in the ones and no digit in the tens places
a could be 3.456 or 2.3, but a could not be 27.34 or 0.56
n is an integer (…, -2,-1,0,1,2,…)

3.2×10^{7} is a big number because 10 is multiplied seven times. Multiplying by 10 moves the decimal point to the right.

$3.2\times 10^{7} = 32{,}000{,}000$ Just move the decimal point 7 times to the right.

2.4×10^{-5} is a small number. $10^{-5} = \frac{1}{10^{5}}$ So, we are dividing by 10 five times. We do this division by 10 moves the decimal point to the left.

$2.4\times 10^{-5} = 0.000024$ Just move the decimal point 5 times to the left.

Examples

1. First write in scientific notation and then simplify: $\dfrac{32,000,000,000,000,000}{160,000,000,000}$

Steps	Reasons
$\dfrac{32{,}000{,}000{,}000{,}000{,}000}{160{,}000{,}000{,}000}$	
$\dfrac{3.2\times10^{16}}{1.6\times10^{11}}$	Rewrite using scientific notation. Notice that 3.2 and 1.6 have a digit in the ones place and no digits in the tens place.
$\dfrac{3.2}{1.6}\times\dfrac{10^{16}}{10^{11}}$	We can split into two fractions or divide the decimal numbers directly and rewrite the 10's using the rule for
$2\times10^{16-11}$	exponents. $\dfrac{x^m}{x^n}=x^{m-n}$
2×10^{5} or $200{,}000$	Write the answer in scientific notation or as a decimal.

2. First write in scientific notation and then simplify: $\dfrac{(72{,}000{,}000)(0.0000000092)}{(0.000036)(1{,}000{,}000{,}000)}$

Steps	Reasons
$\dfrac{(72{,}000{,}000)(0.0000000092)}{(0.000036)(1{,}000{,}000{,}000)}$	
$\dfrac{(7.2\times10^{7})(9.2\times10^{-9})}{(3.6\times10^{-5})(1\times10^{9})}$	Rewrite using scientific notation. Notice that decimals have a digit in the ones place and no digits in the tens place.
$\dfrac{7.2\cdot9.2}{3.6\cdot1}\times\dfrac{10^{7}\cdot10^{-9}}{10^{-5}\cdot10^{9}}$	We can split into two fractions or divide the decimal numbers directly and rewrite the 10's using the rules for exponents:
$18.4\times\dfrac{10^{7+(-9)}}{10^{-5+9}}$	$x^m\cdot x^n=x^{m+n}$ add exponents in the numerator and denominator
$18.4\times\dfrac{10^{-2}}{10^{4}}$	$\dfrac{x^m}{x^n}=x^{m-n}$ subtract exponent in denominator from the exponent in the numerator.
$18.4\times10^{-2-4}$	
$1.84\times10^{1}\times10^{-6}$	To write in scientific notation requires converting the 18.4 to a decimal of 1.84, which leaves us with another 10.
1.84×10^{-5} or 0.0000184	

Exercises

Simplify by using the rules of exponents:

1. 9^2
2. 8^2
3. $(-2)^4$
4. -3^4
5. -2^4
6. $(-3)^4$
7. 5^0
8. -10^0
9. -15^0
10. 6^0
11. $(2^3)^2$
12. $(3^2)^2$
13. $2^3 \cdot 2^2$
14. $3^2 \cdot 3^3$
15. $\frac{9^7}{9^5}$
16. $\frac{4^9}{4^6}$
17. $(4^3)^2$

18. $(5^2)^2$

19. 2^{-4}

20. 6^{-2}

21. -5^{-2}

22. $(-3)^{-2}$

23. $(-4)^{-2}$

24. -4^{-3}

25. -1^{-7}

26. $4^7 \cdot 4^{-9}$

27. $3^5 \cdot 3^{-8}$

28. $\frac{5^8}{5^6}$

29. $\frac{4^9}{4^6}$

30. $\frac{3^7}{3^9}$

31. $\frac{2^5}{2^8}$

32. $3^{-8} \cdot 3^6$

33. $3^{-20} \cdot 3^{17}$

34. $\frac{2^5}{2^7}$

35. $\frac{7^9}{7^{11}}$

Simplify the following using the rules for exponents and leaving no negative exponents. Assume that none of the variables have a value of zero.

36. $x^5 \cdot x^7$

37. $y^{12} \cdot y^{-5}$

38. $z^7 \cdot z^{-11}$

39. $t^{15} \cdot t^{-20}$

40. $\frac{x^5}{x^9}$

41. $\frac{y^7}{y^{15}}$

42. $\frac{x^6 \cdot y^3}{x^2 \cdot y^{11}}$

43. $\frac{x^{10} \cdot y^7}{x^5 \cdot y^3}$

44. $\frac{x^5 \cdot y^{-3}}{x^{-8} \cdot y^7}$

45. $\frac{3^5 \cdot x^5}{3^7 \cdot x^3}$

46. $\frac{2^3 \cdot y^7}{2^6 \cdot y^9}$

47. $\left(\frac{x^5 \cdot y^7}{x^7 \cdot y^9}\right)^{-2}$

48. $\left(\frac{5^8 \cdot t^8}{5^6 \cdot t^{12}}\right)^{-2}$

49. $\left(\frac{3^4 \cdot y^5}{3^6 \cdot y^3}\right)^{-2}$

50. $\left(\frac{x^5 \cdot y^{10}}{x^9 \cdot y^6}\right)^{-2}$

51. $\frac{(x^3 \cdot y^7)^{-2}}{(x^5 \cdot y^6)^{-1}}$

52. $\frac{(x^4 \cdot x^{-5})^{-3}}{(x^{-2} \cdot y^3)^{-2}}$

53. $\frac{(x^{-4} \cdot y^3)^5}{(x^7 \cdot y^{-5})^{-2}}$

Write the decimal number in scientific notation:

54. $49{,}000{,}000{,}000{,}000$

55. $3{,}740{,}000{,}000{,}000$

56. 0.000024

57. 0.000000007624

58. 0.00021

59. $123{,}000{,}000{,}000$

60. $94{,}370{,}000{,}000{,}000$

61. 0.0000062

First convert the decimal numbers to scientific notation and then simplify. You may leave your answer in scientific notation:

62. $156{,}000{,}000{,}000 \cdot 23{,}000{,}000{,}000$

63. $2{,}500{,}000{,}000 \cdot 234{,}000{,}000{,}000$

64. $576{,}000{,}000 \cdot 72{,}000{,}000$

65. $6{,}200{,}000{,}000 \cdot 432{,}000{,}000$

66. $\frac{735{,}000{,}000{,}000}{0.0000021}$

67. $\frac{0.00000682}{22{,}000{,}000{,}000}$

68. $\frac{(3{,}600{,}000)\cdot(0.0000022)}{(0.0012)\cdot(1{,}100{,}000{,}000)}$

69. $\frac{(0.00000028)\cdot(15{,}000{,}000)}{(350{,}000{,}000{,}000{,}000)\cdot(0.00048)}$

70. $\frac{(0.000072)\cdot(360{,}000{,}000)}{(3{,}200{,}000{,}000{,}000)\cdot(0.00081)}$

71. $\frac{(1{,}600{,}000)\cdot(0.00045)}{(0.0000012)\cdot(300{,}000{,}000)}$

For the following two problems, use 9.11×10^{-28} grams for the weight of an electron.

72.. How much do a billion electrons weigh?

73. A googol is a mathematical number with a value of 10^{100}. How much does a googol of electrons weigh?

For the next two problems, a light year is the distance that light travels in a year, which is about 5.88×10^{12} miles.

74. The furthest observed object was a gamma ray burst 13,095,000,000 light years away. How many miles away was this gamma ray burst?

75. The Andromeda galaxy is about 2,500,000 light years away. How many miles away is the Andromeda galaxy?

Exercise Set 1.1

1. $7 < 9$

3. $-8 > -12$

5. $-4 < -3$

7. $12 < |-15|$

9. $|-6| > |4|$

11. $-1, |0|, |-2|, 3, 7, |-11|$

13. $-5, -2, 0, 1, |-6|, |-7|, |11|$

15. -2

17. -21

19. 4

21. -42

23. -7

25. -9

27. -12

29. -5

31. -45

33. -40

35. -63

37. -9

39. -9

41. -7

43. -20

45. 125

47. 16

49. 64

51. -16

53. 17

55. -3

57. -3

59. -28

61. 6

63. -11

65. -9

67. 48

69. -75

71. -304

73. -395 Maria borrows $395.

75. 2528 years

Exercise Set 1.2

1. $2^2 \cdot 3^2$

3. $2^2 \cdot 5^2$

5. $2^2 \cdot 5 \cdot 11$

7. 90

9. 1080

11. 7560

13. 6

15. 56

17. 84

19. 630 students

21. 24 exams

Exercise Set 1.3

1. $\frac{7}{2}$

3. $\frac{53}{8}$

5. $-\frac{39}{7}$

7. $-\frac{107}{7}$

9. $4\frac{3}{5}$

11. $8\frac{5}{9}$

13. $-4\frac{3}{4}$

15. 0.625

17. $9.\overline{3}$

19. 3.2

21. -12.1875
23. $-16.\overline{5}$

25. $32.8\overline{3}$

27. $\frac{57}{100}$

29. $\frac{532}{100}$

31. $\frac{562}{1000}$

33. $\frac{3}{7}$

35. $\frac{5}{6}$

37. $\frac{1}{2}$

39. $\frac{4}{15}$

41. $\frac{1}{9}$

43. $-\frac{4}{21}$

45. $\frac{2}{15}$

47. $12\frac{1}{4}$

49. $-39\frac{5}{27}$

51. $27\frac{1}{2}$

53. $\frac{5}{6}$

55. $-1\frac{1}{2}$

57. $\frac{128}{375}$

59. $-1\frac{1}{3}$

61. $1\frac{11}{16}$

63. $-1\frac{7}{8}$

65. $\frac{1}{3}$

67. $\frac{4}{9}$

69. $\frac{25}{49}$

71. $7\frac{9}{16}$

73. $\frac{5}{7}$

75. $\frac{5}{9}$

77. $\frac{11}{12}$

79. $\frac{19}{60}$

81. $-\frac{8}{15}$

83. $-\frac{67}{75}$

85. $14\frac{1}{2}$

87. $-4\frac{11}{18}$

89. $8\frac{7}{12}$

91. $-2\frac{1}{18}$

93. $-\frac{1}{18}$

95. $-\frac{5}{23}$

97. $\frac{1}{3}$

99. $\frac{37}{60}$

101. $-\frac{7}{20}$

103. $\frac{5}{18}$

105. $1\frac{7}{15}$

107. $-6\frac{11}{14}$

109. $7\frac{3}{5}$ gallons of gas

Exercise Set 1.4

1. 7

3. 8

5. 36

7. 12.530

9. $4{,}488.943$

11. $4\sqrt{2}$

13. $8\sqrt{2}$

15. $15\sqrt{3}$

17. $165\sqrt{2}$

19. $3\sqrt{10}$

21. $10\sqrt{21}$

23. $196\sqrt{3}$

25. $420\sqrt{6}$

27. $5\sqrt{3} + 5$

29. $7\sqrt{2} - 7$

31. $30 + 45\sqrt{3}$

33. $70\sqrt{3} - 56$

35. $9\sqrt{3}$

37. $9\sqrt{5} - 7\sqrt{10}$

39. $13\sqrt{3}$

41. $8\sqrt{2}$

43. a. 792.96 km/hour
    b. 340.05 km/hour
    c. As the depth of the water decreases, the velocity of the Tsunami decreases.

45. 8.82 seconds

<u>Exercise Set 1.5</u>

1. 81

3. 16

5. -16

7. 1

9. -1

11. 64

13. 32

15. 81

17. 4096

19. $\frac{1}{16}$

21. $-\frac{1}{25}$

23. $\frac{1}{16}$

25. -1

27. $\frac{1}{27}$

29. 64

31. $\frac{1}{8}$

33. $\frac{1}{27}$

35. $\frac{1}{49}$

37. y^7

39. $\frac{1}{t^5}$

41. $\frac{1}{y^8}$

43. x^5y^4

45. $\frac{x^2}{9}$

47. x^4y^4

49. $\frac{81}{y^4}$

51. $\frac{1}{xy^8}$

53. $\frac{y^5}{x^6}$

55. $3.74 \text{ x } 10^{12}$

57. $7.624 \text{ x } 10^{-9}$

59. $1.23 \text{ x } 10^{11}$

61. $6.2 \text{ x } 10^{-6}$

63. $5.85 \text{ x } 10^{20}$

65. $2.6784 \text{ x } 10^{18}$

67. $3.1 \text{ x } 10^{-16}$

69. $2.5 \text{ x } 10^{-11}$

71. $2 \text{ x } 10^0$ or 2

73. $9.11 \text{ x } 10^{72}$ grams or $9.11 \text{ x } 10^{69}$ kilograms

75. $1.47 \text{ x } 10^{19}$ miles

Exercise Set 2.1

1. 7

3. -5

5. 57

7. $-2\frac{41}{100}$

9. -84

11. 2

13. $\frac{2}{9}$

15. $-\frac{2}{3}$

17. -45

19. 90

21. -4

23. 75

25. -18

27. a. 71.3 inches
b. The formula overestimates the actual height by 0.3 inches. This trend is not likely to continue indefinitely or men's average height would continually increases. For instance the formula indicates that the average height of men will be about eighteen and a half feet 5000 years from now.

29. a. 18,000 students
b. The formula overestimates the actual number of students by 200.

Exercise Set 2.2

1. -10xy