2

Having mastered the order of operations and signed numbers from Chapter 1 will be very important throughout this chapter. Here we begin a discussion of algebra and the algebraic techniques of simplifying expressions and solving equations. Adult students will recognize these topics and will master them with more or less effort depending on their prior experience and ability. It is not uncommon for adult students to need extra work with the more complicated problems.

Course Outcomes:

- Demonstrate mastery of algebraic skills
- Recognize and apply mathematical concepts to real-world situations

2.1 Algebraic Expressions

The notions of variable and algebraic expression are introduced. Algebraic relations are used to solve real world problems.

2.2 Simplifying Algebraic Expressions

Some properties of real numbers are defined. The idea of algebraic expression is further developed. Like terms are explained for addition and subtraction of algebraic expressions. Students will learn how to simplify complicated algebraic expressions using order of operations.

2.3 Solving Linear Equations

Linear equations are defined. Solving linear equations from simple to complex is explained.

2.4 Literal Equations

Literal equations are defined. Students will be able to solve literal equations.

Algebraic expressions contain numbers variables and arithmetic operations like addition, subtraction, multiplication, division, and exponents. A variable is symbol usually a letter that stands for many numbers or an unknown quantity.

Below algebraic expressions are evaluated for specific numbers by substituting the numbers for the variable. Use parentheses around the number that is being substituted to help avoid making some careless mistakes with operations and negative signs.

Examples

1. Evaluate $5x^2 + 2y$ for the x = 3 and y = 7.

Steps	Reasons
$5(3)^2 + 2(7)$	Substitute the variables with the numbers. Using parentheses, helps us see the appropriate order of operations:
$5(9) + 2(7)$	1. Evaluate exponent.
	2. Multiplication.
$45 + 14$	3. Add
59	

2. Evaluate x + y for x = $3\frac{1}{3}$ and y = $2\frac{4}{5}$.

Steps	Reasons
$3\frac{1}{3} + 2\frac{4}{5}$	Begin with the same substitution.
$\frac{10}{3} + \frac{14}{5}$	Convert the mixed numbers to improper fractions.
$\frac{10}{3}\left(\frac{5}{5}\right) + \frac{14}{5}\left(\frac{3}{3}\right)$	Get the LCD. By multiplying the same number in the numerator and denominator.
$\frac{50}{15} + \frac{42}{15}$	Multiply in the numerator and the denominator.
$\frac{92}{15}$	Add the numerators and keep the LCD.
$\begin{array}{r} 6\frac{2}{15} \\ 15\overline{)92} \\ -\underline{90} \\ 2 \end{array}$	Divide to change the improper fraction to a mixed number.
$6\frac{2}{15}$	Write the answer.

3. Evaluate xyz for x = $-3\frac{1}{2}$, y = $-2\frac{2}{7}$, and z = $-1\frac{3}{4}$

Steps	Reasons
$-3\frac{1}{2}\left(-2\frac{2}{7}\right)\left(-1\frac{3}{4}\right)$	Substitute and write the multiplication. Note the product is negative because there are three negative factors.
$-\frac{7}{2}\cdot\frac{16}{7}\cdot\frac{7}{4}$	Change the mixed numbers to improper fractions. (Multiply whole number by denominator and add the numerator to get the new numerator. Keep old denominator.)
$-\frac{7}{2}\cdot\frac{2\cdot 2\cdot 4}{7}\cdot\frac{7}{4}$	Factor to cancel common factors. Any factor in any numerator cancels with any factor in any denominator.
$-\frac{1}{1}\cdot\frac{2}{1}\cdot\frac{7}{1}$	7´s, 2´s and 4´s cancel. Note that when we cancel a 1 is left.
$-\frac{14}{1}\, or -14$	Multiply numerators and multiply denominators.

4. Evaluate $(x+y)^2 - 2z$ for $x = 5, y =$ – 7, z = –3

Steps	Reasons
$[(5)+(-7)]^2 - 2(-3)$	Replace the variables with the numbers. If the parentheses are used in parentheses are used in replacement, it is easier to determine the operation. The 5 does not need parentheses, but it is still correct to put them there.
$[-2]^2 - 2(-3)$	Work inside parentheses. 5 + (–7) = –2
$4 - 2(-3)$	Exponents: $[-2]^2 = 4$
$4 + 6$	Multiplication: –2(–3) = 6
10	Addition: 4 + 6 = 10

5. Evaluate –x – (–y) for x = –2 and y = 8

Steps	Reason
–(–2) – (–8)	Replace the letter with the number using parentheses.
2 + 8	With two negative signs or minus a negative number, the sign becomes positive or the operation becomes addition.
10	Add the numbers.

6. Evaluate $(x + y)^2 – 3y$ for x = 2 and y = 5.

Steps	Reason
$(2 + 5)^2 – 3(5)$	1. Substitute the variables with the numbers. Using parentheses, helps us see the appropriate operations.
$(7)^2 – 3(5)$	The 2 and 5 in the parentheses do not need parentheses. 2. Perform the operations within parentheses.
$49 – 3(5)$	3. Exponents 4. Multiplication
$49 – 15$	5. Subtraction The answer is 34.
34	

7. Evaluate: $\frac{6x^2yz^3}{(x-y)^2}$ for $x = 4, y = -2, \mathit{and}\ z = -3$

Steps	Reason
$\frac{6x^2yz^3}{(x-y)^2}$	Substitute the variables with the given values for the variables.
$\frac{6(4)^2(-2)(-3)^3}{(4-(-2))^2}$	Use the order of operations to simplify. Treat the numerator (top) and denominator (bottom) separately: In the numerator first do exponents then multiply.
$\frac{6(16)(-2)(-27)}{(4+2)^2}$	In the denominator work inside the parentheses then evaluate the exponent.
$\frac{5184}{6^2}$	
$\frac{5184}{36}$ *or* 144	Simplify the fraction.

Exercises

1. Evaluate $x + y^2$ for $x = 3$ and $y = 2$.
2. Evaluate $x^3 - y^2$ for $x = 5$ and $y = 3$.
3. Evaluate $3x + y^2$ for $x = -3$ and $y = -2$.
4. Evaluate $5x + y^2$ for $x = -4$ and $y = -3$.
5. Evaluate $7x^2 - 3y$ for $x = -3$ and $y = 2$.
6. Evaluate $x + 3y$ for $x = \frac{2}{3}$ and $y = -\frac{1}{2}$.
7. Evaluate $3x - y^2$ for $x = -\frac{3}{4}$ and $y = -\frac{2}{5}$.
8. Evaluate xyz for $x = 4$, $y = -5$, and $z = -2$.
9. Evaluate $2xyz$ for $x = -2$, $y = -3$, and $z = -7$.
10. Evaluate $-3xyz$ for $x = -5$, $y = -1$, and $z = -2$.
11. Evaluate $\frac{5x+2z}{3x-2y}$ for $x = -2$, $y = 3$, and $z = -7$.
12. Evaluate $\frac{3x-2y}{xy}$ for $x = 2$, $y = -1$.
13. Evaluate xyz for $x = -\frac{3}{4}$, $y = \frac{5}{9}$, and $z = -\frac{8}{15}$.
14. Evaluate $-2xyz$ for $x = \frac{5}{12}$, $y = -\frac{9}{25}$, and $z = \frac{10}{27}$.
15. Evaluate $-3xyz$ for $x = -\frac{3}{4}$, $y = \frac{5}{9}$, and $z = -\frac{8}{15}$.
16. Evaluate $4xy^2$ for $x = -2\frac{1}{3}$, and $y = 1\frac{2}{3}$.
17. Evaluate $2x^2y$ for $x = 2\frac{1}{2}$, $y = -3\frac{3}{5}$.
18. Evaluate $4x^2y^2 - 2xy + 3x^2y^2$ for $x = 4$, $y = 5$.

19. Evaluate $2x^2y^2 + xy + 3y^2$ for $x = 3$, $y = 2$.

20. Evaluate $\frac{2xyz}{2x-y-z}$ for $x = -5$, $y = 2$, and $z = -4$.

21. Evaluate $\frac{3x-2y}{xy}$ for $x = 2$, $y = -1$.

22. Evaluate $x^2y^2 - 5xy + x^2y^2$ for $x = -2$, $y = -3$.

23. Evaluate $5x^2y^2 - 2xy + 4x^2y^2$ for $x = -3$, $y = -1$.

24. Evaluate $3x^2y^3 - 2xy - x^3y^2$ for $x = -2$, $y = 3$.

25. Evaluate $x^2y^2 - 3xy - 2x^2y^2$ for $x = -1$, $y = -3$.

26. The cost of a house in dollars can be estimated by the formula $C = 7100x + 27{,}500$ where x is the number of years after 1970.
a. Use the formula to estimate the cost of a house in 2005.
b. If the actual cost of a house in 2005 was $280,000, does the formula underestimate or overestimate the actual price? By how much?

27. People's heights have been increasing over the last three hundred years. The formula $H = .03t + 62$ can be used to estimate men's average heights in inches t years after 1700.
a. Use the formula to estimate men's average height in the year 2010.
b. If the actual average height of men is 71 inches, does the formula underestimate or overestimate the actual height? By how much? Do you think that this trend will continue indefinitely?

28. The cost of a new car in dollars can be estimated by the formula $C = -1.8x^2 + 975x + 9500$ where x is the number of years after 1990.
a. Use the formula to estimate the cost of a new car in 2015.
b. If the actual cost of a new car in 2015 was $31,950, does the formula underestimate or overestimate the actual price? By how much?

29. The number of students at a large university can be estimated by the formula $N = -5x^2 + 800x + 4000$ where x is the number of years after 1980.
a. Use the formula to estimate the number of students in the year 2000.

b. If there were 17,800 students in the year 2000, does the formula underestimate or overestimate the actual number of students? By how much?

Many of the properties of real numbers we know even if we do not know their name. For instance, the Multiplication Property of Zero tells us that if we multiply by zero we get zero. We knew that! We need to know the names of the properties in order to communicate why we are able to perform certain steps. The Commutative property, Associative Property, and Distributive Property are used frequently when simplifying algebraic expressions.

The Commutative Property states we can add or multiply in either order.

$a + b = b + a$ addition

$a \cdot b = b \cdot a$ multiplication

For example, 5+3=3+5 shows commutative property of addition

The Associative Property allows us to change the grouping if we have all addition or all multiplication.

$(a + b) + c = a + (b + c)$ addition

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ multiplication

An example showing the commutative property of multiplication:

$$(2\cdot 3)\cdot 4 = 2\cdot(3\cdot 4)$$
$$(6)\cdot 4 = 2\cdot(12)$$
$$24 = 24$$

Between the Commutative Property and the Associative Property:

1. If there is only addition, we can add in any order. Subtraction can be thought of as adding a negative number.
2. If there is only multiplication we can multiply in any order.

It is especially useful to think of these two properties like this when we have variables:

1. $2 + 3x + 5 + 7x = 7 + 10x$ I just add the numbers and add the variables.
2. $(-3)x(-2) = 6x$ I just multiply the numbers. I can change the order mentally.

The Distributive Property lets us change the order of operations. We multiply outside parentheses before doing addition or subtraction inside parentheses.

$2\,(3 + 5) = 2\cdot 3 + 2\cdot 5$ Here we use the distributive property. Both 3 and 5 are multiplied by the 2 outside of parentheses

We see that the distributive property works by simplifying both sides.

$2\,(3 + 5)$	$2\cdot 3 + 2\cdot 5$	
$2(8)$	$6+10$	
16	16	Both sides are the same.

The Distributive Property is important with variables because we cannot always do the addition or subtraction in the parentheses because we do not always have like terms.

3(x + 5) = 3·x + 3·5	We cannot add x + 5 in parentheses because they are not like terms; however, we can do the multiplication by the distributive property.
= 3x + 15	Here we have simplified the expression.

Examples

1. Simplify $3x(-5)$

$3x(-5) = -15x$ Done. The answer is –15x.

We want to do this problem in one step. Just multiply the numbers and keep the variable. Because it is all multiplication, we can multiply in any order. We multiply the numbers and get -15. Why does this work?

$3x(-5) = 3(-5)x$	Use the commutative property to change the order for multiplication.
$= -15x$	$3(-5) = -15$

2. Simplify $\left(-\frac{2}{3}\right)x\left(-\frac{3}{2}\right)$

$\left(-\frac{2}{3}\right)x\left(-\frac{3}{2}\right) = x$ Done. The answer is x.

Multiply the numbers $\left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right) = 1$. We get $1x$ or x. The key to working with variable is focusing on the appropriate arithmetic. We can multiply the original expression in any order because it as all multiplication (commutative property). Just multiply the numbers and keep the variables.

3. Simplify $9 - 15m + 15m$

$9 - 15m + 15m = 9$ Done. The answer is 9.

$-15m + 15m = 0$ We are adding the same numbers but with opposite signs.

4. Simplify $3(2x-5y+6)$

$$\begin{aligned}3(2x-5y+6) &= 3\cdot 2x-3\cdot 5y+3\cdot 6 \\ &= 6x-15y+18\end{aligned}$$

Distributive Property

We can do this problem in one step.

This problem can be done in one step because the arithmetic is easy. We always need to be careful with our signs and multiply each term inside the parentheses by the two which is outside the parentheses.

5. Simplify $-5(5m-7n-11)$

$-5(5m-7n-11) = -25m+35n+55$ **Done. The answer is –25m+ 35n + 55.**

Multiply each number inside parentheses by the -5 outside parentheses. The signs can be tricky at first. Just remember when multiplying, two negative factors yield a positive product. If only one factor is positive, then the product is negative.

The missing steps are as follows:

$$\begin{aligned}-5(5m-7n-11) &= -5(5m)-(-5)7n-(-5)11 \\ &= -25m+35n+55\end{aligned}$$

Distributive Property

Multiply numbers. Careful with signs.

6. Rewrite $-(6s-5t-36)$ without parentheses.

$-(6s-5t-36) = -6s+5t+36$ Done. The answer is –6s + 5t + 36.

When there is a negative sign outside of parentheses change all the signs inside the parentheses. Easy, remember it, done.

Lets look at the steps to see why it is true. In practice, we just change all the signs in one step like we did above.

$-(6s-5t-36) = -1(6s-5t-36)$ –3 = –1·3. So, we can replace the negative sign with negative one.

$(-1)(6s)-(-1)(5t)-(-1)(36)$ Distributive property

$-6s+5t+36$ Multiply and – (–) becomes +

For collecting like terms, the variable part must be exactly the same.

3**x** + 2**x** = 5**x** We add the numbers and keep the variable part.

7**x²y** – 4**x²y** = 3**x²y** Because the variable part is exactly the same, we subtract the numbers and keep the same variable part.

12**x** – 5**y** We stop. We cannot go further because the variable part is not the same.

6**xy²** – 3**x²y** We stop. We cannot go further because the variable part is not the same. The exponents are with different variables.

11**x²** + 3**x** We stop. We cannot go further because the variable part is not the same. One of the x's has exponent 2 and the other does not.

The key to adding and subtracting with variables is that the variable part must be exactly the same. If the variable part is not the same, then we cannot add or subtract. Also, the variable part stays the same when we add and subtract.

Examples

7. Simplify $12t^2+5t-7+11t^2-8t+9$

Steps	Reasons
$12t^2+5t-7+11t^2-8t+9$	For t^2, $12t^2+11t^2=23t^2$ For t, $5t-8t=-3t$ For the numbers, $-7+9=2$
$23t^2-3t+2$	
	Done. The answer is $23t^2-3t+2$.

This problem should be done in one step. Add or subtract the numbers in front of the t^2, t, and the numbers. We can add the numbers in any order because of the Commutative Property. Even the subtraction can be thought of as addition by adding a negative number.

8. Simplify $5x-3(2x+6)+4(7x-8)$

Steps	Reasons
$5x-3(2x+6)+4(7x-8)$	Nothing can be done in parentheses.
$5x-6x-18+28x-32$	Use the distributive property to get rid of parentheses. Here distribute –3 and 4.
$27x-50$	Collect like terms.

9. Simplify: $12z^2 - 5[3(2z-4) - 6(z^2+1)]$

Steps	Reasons
	Start inside the brackets.
$12z^2 - 5\ [3\ (2z-4) - 6\ (z^2+1)]$	Distribute the 3 and the – 6.
$12z^2 - 5\ [\ 6z - 12 - 6z^2 - 6\ ]$	
$12z^2 - 5\ [\ 6z - 18 - 6z^2\ ]$	Collect like terms in the brackets. $-12 - 6 = -18$
$12z^2 - 30z + 90 + 30z^2$	Distribute the –5.
$42z^2 - 30z + 90$	Collect the like terms. $12z^2 + 30z^2 = 42z^2$

10. Add: $(3x^2 - 2x + 5) + (7x^2 + 2x - 4)$

Steps	Reasons
$(3x^2 - 2x + 5) + (7x^2 + 2x - 4)$	To add the two expressions just collect like terms.
$3x^2 - 2x + 5 + 7x^2 + 2x - 4$	Drop the parentheses because there is nothing to distribute.
$10x^2 + 1$	Collect like terms: $3x^2 + 7x^2 = 10x^2$ $-2x + 2x = 0$ $5 - 4 = 1$

11. Subtract: $(8x^3 - 5x + 11) - (4x^3 - 6x^2 + 5x + 11)$

Steps	Reasons
$(8x^3 - 5x + 11) - (4x^3 - 6x^2 + 5x + 11)$	Before subtracting we must change all the signs in the parentheses after subtraction.
$8x^3 - 5x + 11 - 4x^3 + 6x^2 - 5x - 11$	Now we collect like terms.
$4x^3 + 6x^2 - 10x$	Collect like terms: $8x^3 - 4x^3 = 4x^3$ $6x^2$ $-5x - 5x = -10x$ $11 - 11 = 0$

Exercises

Simplify the algebraic expression:

1. $-5 \cdot x \cdot y \cdot 2$

2. $7 \cdot t \cdot z \cdot (-3)$

3. $\left(\frac{3}{5}\right) xy \left(\frac{5}{3}\right)$

4. $\left(-\frac{4}{7}\right) rs \left(\frac{7}{4}\right)$

5. $12 - 2x + 5x$

6. $7x + 3 - 2x + 4$

7. $15 - 3y - 7y - 25$

8. $0.25x - 3.21 + 2.45x + 5.2$

9. $3.21t - 5.35 - 4.76t - 3.27$

10. $3x - 5y + 4z - 2x - 7y - 8z$

11. $5 - 3x + 2y - 7 + 8x - 9y$

12. $-1.32x + 0.32y - 3.83 + 5.21x - 5.31y - 2.73$

13. $4.21x - 9.8y - 12.76 - (-2.13x) + 12.3y - 5.35$

14. $3(2x - 5y)$

15. $4(5x - 2y + 3)$

16. $-3(3x + 2y - 4)$

17. $-5(4x - 3y - 8)$

18. $-(9x - 12y - 5)$

19. $-(2x - 7y + 8)$

20. $3x + 4x - 3y + 2y$

21. $-5x + 8x - 7y - 8y$

22. $12m - 3m + 8n - 15n$

23. $7s - 15s - 11t - 3t$

24. $5x^2 - 3x^2 - 5x + 4x$

25. $9x - 5x^2 + 2x - 3x^2$

26. $5x - 8x^2 + 3x - 10x^2$

27. $7x^2 + 2x^2 - 9x - 3x$

28. $-7xy^2 + 8x^2y - 9xy^2 + 5x^2y$

29. $12x^2y - 3xy^2 - 15x^2y + 10xy^2$

30. $12x + 5 + 2(3x - 7)$

31. $5x - 3 - 4(2x - 8)$

32. $7x - 8 - 3(5x - 4)$

33. $3x - 4 + 5(2x + 1)$

34. $8x - 3 - (3x - 5)$

35. $7x + 4 - (2x - 7)$

36. $4x + 5 - (4x + 1)$

37. $3x - 9 - (2x - 5)$

38. $12x - 9 - (5x - 7) + 4$

39. $15x - 12 - (4x + 9) - 8$

40. $(4x^2 + 2x - 3) + (3x^2 - 4x - 3)$

41. $(2x^2 - 3x + 8) + (2x^2 + 3x - 6)$

42. $(3x^2 - 5x - 7) - (2x^2 + 4x - 9)$

43. $(7x^2 - 3x + 2) - (5x^2 - 3x + 8)$

44. $(4x^2 - 3x + 2) + (x^2 + x - 3)$

45. $(3x^2 + 2x - 5) + (x^2 - 7)$

46. $(2x^2 - 3x) - (x^2 - 5x - 4)$

47. $(5x^2 - 3x - 9) - (x^2 - 5x - 9)$

48. $(4x^2 - 3x + 8) + (2x^2 + 4x - 11)$

49. $(3x^2 - 8x - 9) + (4x^2 + 8x - 9)$

50. $7 - 2[3(2x - 5) - (5x + 3)]$

51. $5 - 3[2(3x + 1) - (2x - 3^2)]$

52. $6+4[-2(5x + 2^2) + 3(7x - 5^2)]$

53. $8 + 2[-4(4x - 3^2) + 2(3x - 2^2)]$

54. $5 - 3[3(2x - 3^2) - (4x - 2^2)]$

55. $4 - 5[2(5x - 4^2) - (12x - 3^2)]$

56. $2\{5 - [3(2x - 2^2) - (-3x) + 2(5x - 2^2)]\}$

57. $3 - 2\{7x - [-2(3x - 1) + 3(5x - 3^2)]\}$

58. $5t^2 - 3\{-4t^2 + 2[(3t^2 - 2t + 1) - (5t^2 - 2t - 3^2)]\}$

59. $4y^2 - 3\{2y^2 - [(4y^2 - y + 2) - (3y^2 + 2y - 5)]\}$

A **linear equation** can be written in the form ax+b=0 where a,b are numbers and x is a variable. Many times we will have to simplify before solving these equations, but we should recognize that there is one variable, which may appear more than once, the variable does not have an exponent (nor is it under a root), and the variable is not in the denominator.

Solutions to equations are any true replacement for the variable. We can check to see if $x=3$ is a solution to the equation $5x-2=3x+4$ by replacing the x's with 3 in the equation.

$5x-2=3x+4$	Substitute x with 3 and do the arithmetic.
$5(3)-2=3(3)+4$	
$15-2=9+4$	Since we end up with a true statement $13=13$, $x=3$ is a solution to the equation $5x-2=3x+4$.
$13=13$	

At the heart of solving equations are two basic properties:
1. We can add or subtract the same number from both sides of an equation.
2. We can multiply and divide the same number on both sides of an equation.

When solving these linear equations, the objective is to isolate the variable.

The basic steps:
1. Determine how the variable is connected to the number.
2. Perform the opposite operation on both sides of the equation.
3. Check by replacing the solution for the variable in the original equation.

<u>Examples</u>

1. Solve $x+15=34$

<u>Steps</u>	<u>Reasons</u>
$x+15=34$	To solve the equation isolate the variable *x*.
$\;\;-15\quad-15$	Because *x* added to 15*,* subtract 15 from both sides. Since 15 – 15 = 0, the *x* is left alone.
$x=19$	Here the equation is solved. The variable appears alone.

Notice that the 15 went to the other side by changing the + 15 to – 15.

To check replace the solution in the original equation. $19+15=34$ is a true statement.

2. Solve $4x = 36$

Steps	Reasons
$\frac{4x}{4} = \frac{36}{4}$	The variable *x* needs to be isolated. Because the 4 is connected to the *x* by multiplication, both sides of the equation are divided by 4.
$x = 9$	The variable is isolated and the equation is solved.

Check: 4(9) = 36 is true. In the end we just divided on the right and got rid of the 4 on the left.

3. Solve $-4x = 32$

Steps	Reasons
$-4x = 32$	The –4 is connected to the variable by multiplication. So, divide both sides by –4.
$\frac{-4x}{-4} = \frac{32}{-4}$	Divide both sides by –4. The x will be left alone because (–4) ÷ (–4) = 1 and 1x is the same as x.
$x = -8$	All done.

Check your answer by putting –8 into the original equation:
$-4(-8) = 32$ is true.

The numbers may be fractions. You should still determine how the number is connected to the variable and perform the opposite operation

4. Solve $\frac{2}{3}x = 6$

Steps	Reasons
$\frac{2}{3}x = 6$	We want to get the variable by itself. The fraction is connected to the variable by multiplication. We should divide both sides by $\frac{2}{3}$.
$\frac{3}{2} \cdot \frac{2}{3}x = 6 \cdot \frac{3}{2}$	Dividing by $\frac{2}{3}$ is the same as multiplying by $\frac{3}{2}$.
$x = \frac{6}{1} \cdot \frac{3}{2}$	Write 6 as an improper fraction.
$x = 9$	Cancel the common factor.

5. Solve $-\frac{3}{5}=t+\frac{1}{2}$

Steps	Reasons
$-\frac{3}{5}=t+\frac{1}{2}$	We want to get the variable by itself. The fraction is connected to the variable by addition. We should subtract both sides by $\frac{1}{2}$.
$-\frac{3}{5}=t+\frac{1}{2}$ $-\frac{1}{2} \quad -\frac{1}{2}$	Subtracting on the right yields t. To subtract on the left requires more steps. Go to the side and do the necessary steps. $-\frac{3}{5}-\frac{1}{2}=-\frac{3}{5}\cdot\frac{2}{2}-\frac{1}{2}\cdot\frac{5}{5}=\frac{-6}{10}-\frac{5}{10}=\frac{-11}{10}\, or\, -1\frac{1}{10}$
$t=-1\frac{1}{10}$	Write your answer with the variable on the left.

Here we are starting to solve more complicated equations of the form ax + b = c. The idea is still to get x alone. Now we will need to perform two operations in order to get the x alone.

Steps:
1. Add or subtract from both sides.
2. Multiply or divide both sides.

Note: We tend to add and subtract first. This is different from the order of operations. We do not want to divide first because we may introduce more fractions.

Examples

6. Solve: $3x-5=7$

Steps	Reasons
$3x-5=7$	Get the x alone. Notice if we divide both sides by 3 we are going to get fractions.
$3x-5=7$ $+5 \quad +5$	Add 5 to both sides.
$3x=12$	Do the addition.
$\frac{3x}{3}=\frac{12}{3}$	3 is connected to the variable by multiplication. So divide both sides by 3.
$x=4$	Do the division to get the solution.

Check:
Replace the variable in the original equation with the solution. Use parentheses

$3(4)-5=7$

$12-5=7$

$7=7$

7. Solve: $8=5+6z$

Steps	Reasons
$8=5+6z$ $-5\quad -5$	Get the z alone. Notice if we divide both sides by 6 we are going to get fractions. So, begin by subtracting 5 from both sides.
$3=6z$	Do the subtraction.
$\frac{3}{6}=\frac{6z}{6}$	6 is connected to the variable by multiplication. So divide both sides by 6.
$\frac{1}{2}=z$ or	Simplify the fraction.
$z=\frac{1}{2}$	Write the answer with the variable on the left.

8. Solve: $\frac{2}{3}x-\frac{4}{5}=-\frac{3}{4}$

Steps	Reasons
$\frac{2}{3}x-\frac{4}{5}=-\frac{3}{4}$	Get the x alone. We still need to add and multiply both sides, but this time there are fractions.
$\frac{2}{3}x-\frac{4}{5}=-\frac{3}{4}$ $+\frac{4}{5}\quad +\frac{4}{5}$	$\frac{4}{5}$ is subtracted. So add $\frac{4}{5}$ to both sides of the equation.
$\frac{2}{3}x=\frac{1}{20}$	Add the fractions on the side: $-\frac{3}{4}+\frac{4}{5}=-\frac{3}{4}\cdot\frac{5}{5}+\frac{4}{5}\cdot\frac{4}{4}=\frac{-15}{20}+\frac{16}{20}=\frac{1}{20}$
$\frac{3}{2}\cdot\frac{2}{3}x=\frac{1}{20}\cdot\frac{3}{2}$	Instead of dividing by $\frac{2}{3}$ we save a step by immediately multiplying by $\frac{3}{2}$.
$x=\frac{3}{40}$	Simplify by doing the multiplication.

Check:

Replace the variable in the original equation with the solution. Use parentheses

$$\frac{2}{3}\left(\frac{3}{40}\right)-\frac{4}{5}=-\frac{3}{4}$$
$$\frac{1}{20}-\frac{4}{5}=-\frac{3}{4}$$
$$\frac{1}{20}-\frac{4}{5}\cdot\frac{4}{4}=-\frac{3}{4}$$
$$\frac{1}{20}-\frac{16}{20}=-\frac{3}{4}$$
$$-\frac{15}{20}=-\frac{3}{4}$$
$$-\frac{3}{4}=-\frac{3}{4}$$

Same answer both sides checks.

9. Solve: $7x-15-10x=6$

Steps	Reasons
$7x-15-10x=6$	Here we can simplify the left before we start.
$-3x-15=6$	Collect like terms.
$\begin{array}{r}-3x-15=6\\ +15\ \ +15\end{array}$	15 is subtracted. So add 15 to both sides.
$-3x=21$	Add.
$\frac{-3x}{-3}=\frac{21}{-3}$	–3 is multiplied by x. So divide both sides by –3.
$x=-7$	Do the division.

To check replace the variable in the original equation with the solution. Use parentheses

$$7(-7)-15-10(-7)=6$$
$$-49-15+70=6$$
$$-64+70=6$$
$$6=6$$

When working with more complicated equations, we may need to simplify one or both sides of the equation by collecting like terms.

10. Solve: $5x-1+2x=4x+8$

Steps	Reasons
$5x-1+2x=4x+8$	Start by collecting like terms on the left hand side of the equation.
$7x-1=4x+8$	Adding variables: $5x+2x=7x$
$\begin{array}{rcl} 7x-1 &=& 4x+8 \\ -4x & & -4x \end{array}$	I chose to get variables on the left. So I get rid of the 4x on the right by subtracting it from both sides.
$3x-1=8$	Subtracting variables: $7x-4x=3x$
$\begin{array}{rcl} 3x-1 &=& 8 \\ +1 & & +1 \end{array}$	Now it is like section 6.2 Get the numbers on the right. 1 is subtracted. So we add 1 to both sides.
$3x=9$	Add.
$\frac{3x}{3}=\frac{9}{3}$	Because x is connected to the variable by multiplication we divide both sides by 3.
$x=3$	Divide.

Check:
Replace the variable in the original equation with the solution. Use parentheses

$$5(3)-1+2(3)=4(3)+8$$
$$15-1+6=12+8$$
$$20=20$$

<u>These example demonstrates all the possible steps for solving linear equations:</u>
1. Simplify both sides of the equation by collecting like terms.
2. Get all the variables on one side of the equation and all the numbers on the other side of the equation. Here we do the opposite operation of what we see.
3. Get the variable alone by dividing both sides of the equation by the number in front of the variable.

Sometimes students confuse steps 1 and 2. In step 1 we treat each side separately and perform the operation as we see it. In step 2, we perform the opposite operation of what we see to get the variables or numbers to the other side.

11. Solve: $3-5(x+3)=3(x+2)+14$

Steps	Reasons
$3-5(x+3)=3(x+2)+14$	Simplify both sides by performing using the order of operations.
$3-5x-15=3x+6+14$	Multiply using the distributive property to get rid of the parentheses on both sides.
$-12-5x=3x+20$	Add and subtract the like terms on each side.
$\begin{array}{l} -12-5x=3x+20 \\ +12 \qquad\qquad +12 \end{array}$	Here I decided to get the variables on the left and the numbers on the right. Now I am doing the opposite operation. The –12 becomes + 12. We use the opposite operation to solve equations.
$-5x=3x+32$	Add on both sides.
$\begin{array}{l} -5x=3x+32 \\ -3x \quad -3x \end{array}$	Get the variables on the left by using the opposite. 3x is positive. So subtract 3x from both sides.
$-8x=32$	Subtract on both sides.
$\dfrac{-8x}{-8}=\dfrac{32}{-8}$	–8 is connected to x by multiplication. Divide both sides by –8.
$x=-4$	Divide.

Check:

Replace the variable in the original equation with the solution.

$$
\begin{aligned}
3-5(-4+3)&=3(-4+2)+14 \\
3-5(-1)&=3(-2)+14 \\
3+5&=-6+14 \\
8&=8
\end{aligned}
$$

12. Solve: $2x-3(2x+1)=2(3-5x)+9$

<u>Steps</u>	<u>Reasons</u>
$2x-3(2x+1)=2(3-5x)+9$	Simplify both sides by performing using the order of operations.
$2x-6x-3=6-10x+9$	Multiply using the distributive property to get rid of the parentheses on both sides.
$-4x-3=-10x+15$	Add and subtract the like terms on each side.
$\begin{array}{l} -4x-3=-10x+15 \\ +10x \qquad\quad +10x \end{array}$	Here I decided to get the variables on the left and the numbers on the right. Now I am doing the opposite operation. The –10x becomes + 10x. We use the opposite operation to solve equations.
$6x-3=15$	Add on both sides.
$\begin{array}{l} 6x-3=15 \\ \quad +3 \quad +3 \end{array}$	Get the number on the right by using the opposite. 3 is subtracted. So add 3 to both sides.
$6x=18$	Add on both sides.
$\frac{6x}{6}=\frac{18}{6}$	6 is connected to x by multiplication. Divide both sides by 6.
$x=3$	Divide.

Check:

Replace the variable in the original equation with the solution. Use parentheses

$$2(3)-3(2(3)+1)=2(3-5(3))+9$$
$$2(3)-3(6+1)=2(3-15)+9$$
$$2(3)-3(7)=2(-12)+9$$
$$6-21=-24+9$$
$$-15=-15$$

Exercises

Solve and check.

1. $x - 3 = 7$

2. $x + 5 = 11$

3. $x + 9 = -5$

4. $x - 12 = -15$

5. $x - \frac{2}{3} = \frac{3}{4}$

6. $x + \frac{1}{2} = \frac{3}{5}$

7. $x + 0.25 = 0.13$

8. $x - 1.34 = -2.19$

9. $x + 2\frac{3}{4} = 1\frac{7}{8}$

10. $x - 3\frac{1}{3} = -4\frac{5}{6}$

11. $5x = -35$

12. $3x = -21$

13. $-4x = -16$

14. $-6x = -18$

15. $4.2x = -14.7$

16. $-6.4x = -4.8$

17. $\frac{2}{3}x = -\frac{4}{7}$

18. $-\frac{3}{4}x = -\frac{9}{16}$

19. $4x - 3 = 13$

20. $2x + 9 = 17$

21. $3x - 7 = -19$

22. $5x + 16 = -14$

23. $7 - 2x = 3$

24. $15 - 4x = -9$

25. $-11 - 3x = 4$

26. $14 - 6x = -10$

27. $3(x - 4) = 5(x + 4)$

28. $4(3x - 3) = 2(2x + 6)$

29. $5(2x + 3) = 3(2x + 4) - 13$

30. $-2(3x - 5) = 4(x - 5)$

31. $-3(x - 5) = 6 - 4(2x - 1)$

32. $5 - (x - 3) = 16 - 3(x + 4)$

33. $11 - (3x - 7) = 12 - 5(x + 2)$

34. $14 - 2(3x + 6) = 3x - (2x - 16)$

35. $27 - 3(x + 4) = 4x - (2x - 20)$

36. $15 - (3x - 8) = 4x - 5(3x - 7) - 25$

37. $11 - [2x - (-7)] = -6 - 4(3x - 8)$

38. $16 - [4x - (-5) - 8] = -5[2x - (-7) - 5] - 9$

39. $2[x-(-3x)-(-5)-7]=-3[2x-11-(-7)]+8$

40. $-5[2x-(-4x)-(-3)+7]=-3[2-5x-(-3x)-7]+12x$

Literal equations have more than one variable. Literal equations may be formulas in one form that we want to manipulate to get into another form. The key is to focus on the variable that you are solving for and get everything else on the other side as if you were solving an equation with one variable.

Examples

1. Solve $P = 2L + 2W$ for W.

Steps	Reasons
$P = 2L + 2W$	We want W alone:
$-2L \quad -2L$	1. Get the 2L on the other side by subtracting it from both sides.
$P - 2L = 2W$	
$\dfrac{P-2L}{2} = \dfrac{2W}{2}$	2. Divide both sides by 2 to get W by itself.
$W = \dfrac{P-2L}{2}$	

2. Solve $2x - 5y = 20$ for y.

Steps	Reasons
$2x - 5y = 20$	We want y alone:
$-5y = -2x + 20$	1. Get the 2x on the other side by subtracting it from both sides.
$\dfrac{-5y}{-5} = \dfrac{-2x+20}{-5}$	2. Divide both sides by –5 to get y by itself.
$y = \dfrac{-2x}{-5} + \dfrac{20}{-5}$	3. Simplify the fractions.
$y = \dfrac{2}{5}x - 4$	

3. Solve $A = P + Pmt$ for t

Steps	Reasons
$A = P + Pmt$	
$A - P = Pmt$	We want to solve for t. First we need to subtract P from both sides.
$\frac{A-P}{Pm} = \frac{Pmt}{Pm}$	Next divide both sides by Pm.
$\frac{A-P}{Pm} = t$	We get t alone on the right.
$t = \frac{A-P}{Pm}$	Generally we write our answer with the variable we are solving for on the left.

4. Solve $A = \frac{x+y+z}{3}$ for x.

Steps	Reasons
$A = \frac{x+y+z}{3}$	If there is a number or variable in the denominator, clear fractions.
$3 \cdot A = \frac{x+y+z}{\cancel{3}} \cdot \frac{\cancel{3}}{1}$	1. Multiply both sides by the denominator 3.
$3A = x + y + z$	2. Subtract y and z from both sides to get the x alone.
$3A - y - z = x$	
$x = 3a - y - z$	

5. Solve $A = \frac{t}{1-s}$ for s.

Steps	Reasons
$A = \frac{t}{1-s}$	We want s alone:
$A(1-s) = \frac{t}{\cancel{1-s}} \cdot \frac{\cancel{1-s}}{1}$	1. Clear fractions by multiplying both sides by $1-s$.
$A - As = t$ $A - t = As$	2. Get the s term on one side. Notice that miving the terms to the other side changes the sign. We are just adding As to both sides and subtracting t from both sides.
$\frac{A-t}{A} = \frac{\cancel{A}s}{\cancel{A}}$	3. Cancel A by dividing both sides by A.
$s = \frac{A-t}{A}$	That is it.

Exercises

Solve the following literal equations for the given variable.

1. Solve $A = L \cdot W$ for L.
2. Solve $A = L \cdot W$ for W.
3. Solve $P = 2L + 2W$ for L.
4. Solve $5x + 3y = 45$ for x.
5. Solve $-3x + 4y = -24$ for x
6. Solve $2x - 3y = -6$ for y.
7. Solve $4x - 7y = 28$ for y.
8. Solve $A = P + Pmt$ for m.
9. Solve $A = \frac{x+y+z}{3}$ for y.
10. Solve $A = \frac{x+y+z}{3}$ for z.
11. Solve $M = \frac{t}{r+s}$ for r.
12. Solve $M = \frac{t}{r+s}$ for s.
13. Solve $F = \frac{9}{5}C + 32$ for C.
14. Solve $A = P(1 + rt)$ for r.
15. Solve $A = P(1 + rt)$ for t.

3

Here we focus on applied problems or the "dreaded word problem." An overall method is discussed to give students an approach. Using individual strategies for specific types of applied problems is the key to becoming a good algebraic problem solver. Keep at it and try to determine what "type" of problem is presented.

Course Outcomes:

- Demonstrate mastery of algebraic skills
- Recognize and apply mathematical concepts to real-world situations

3.1 Translation Problems

The arithmetic operations for specific phrases are introduced. The concepts of expression and equation are developed. Students learn to translate phrases into algebraic expressions and algebraic equations by focusing on key words.

3.2 General Strategy for Problem Solving

A general approach to solving application problems is outlined. Students learn to identify specific types of problems and use appropriate methods for them.

3.3 Ratios and Solving Proportions

Rates, ratios, and proportions are defined. Students learn to solve proportions. Students will recognize certain applied problems as proportions.

3.4 Percents

This section begins with a discussion of changing between percents and fractions or decimal numbers. The basic percent equation, mark-up, and mark down are covered. This section may be covered later just prior to Chapter 6.

3.5 Inequalities

Solving inequalities and compound inequalities is discussed. Graphing inequalities in one variable and expressing answers in set-builder notation are explained. Applied problems for inequalities are covered.

Translating words to variable expressions or equations is made easier by knowing the phrases that indicate the various operations and breaking the words down to specific parts. The textbook highlights in blue phrases that indicate the different operations.

Addition

Phrase	Example	Operation
added to	7 is added to 12	7+12
more than	3 more than 8	8+3
the sum of	the sum of 6 and 7	6+7
increased by	9 increased by 8	9+8
the total of	the total of 11 and 15	11+15
plus	2 plus 4	2+4

Subtraction

Phrase	Example	Operation
minus	11 minus 7	11 – 7
less	9 less 5	9 – 5
less than	3 less than 6	6 – 3
the difference between	the difference between 15 and 8	15 – 8
decreased by	20 decreased by 14	20 – 14
subtract…from	subtract 2 from 30	30 – 2

Multiplication

Phrase	Example	Operation
times	12 times 7	$12 \cdot 7$
the product of	the product of 11 and 6	$11 \cdot 6$
multiplied by	10 multiplied by 3	$10 \cdot 3$
twice	twice 5	$2 \cdot 5$
a fraction of	$\frac{1}{3}$ of 15	$\frac{1}{3} \cdot 15$
a percent of	20% of 30	$0.2 \cdot 30$

Division

Phrase	Example	Operation
the quotient of	the quotient of 40 and 8	40÷8
divided by	12 divided by 3	12÷3

Equality

Phrase	Example	Operation
is	The quotient of 36 and 4 **is** 9.	36 ÷4 = 9
equals	The difference between 30 and 11 **equals** 19.	30 –11 = 19
was	10 minus 4 **was** 6.	10 – 4 = 6
represents	25 increased by 14 **represents** 39.	25 + 14 = 39
is the same as	The product of 7 and 8 **is the same as** 56.	7 · 8 = 56
is equal to	15 more than 20 **is equal to** 35.	20 + 15 = 35

Power

Phrase	Example	Operation
the square of	the square of y	y^2
the second power of	the second power of y	y^2
the cube of	the cube of t	t^3
the third power of	the third power of t	t^3

Variable expressions do not have an equality sign. (Variable equations have an equality sign.) Do not write an equality sign if the directions ask for a variable expression.

The key to translating words to expressions is breaking down the words into smaller parts:
1. Write the phrase in one long line.
2. Translate words to numbers right away (fifteen is 15)
3. Identify the operations.
4. Often "and" separates two parts. Replace "and" with the appropriate operation.

Examples

1. Translate into a variable expression: “the sum of twice x and fifteen”

Steps	Reasons
the sum of twice x and 15	Write the phrase as one long line. Translate the number.
twice x + 15	Sum indicates addition at the "and"
$2x + 15$	Twice means multiply by 2.

2. Translate into a variable expression and simplify:
“fifteen less than the difference between a number and six”

Steps	Reasons
15 less than the difference between a number and 6	Write the phrase as one long line. Translate the numbers.
15 less than x – 6	Difference indicates subtraction. Replace "and" with subtraction. Replace "a number" with your favorite letter.
$(x - 6) - 15$	"less than" indicates subtraction.
$x - 6 - 15$	The directions also say to simplify.
$x - 21$	Now it is done.

3. Translate into a variable expression and simplify:
"the difference between a number and four added to the sum of the number and eight"

Steps	Reasons
the difference between a number and 4 added to the sum of the number and 8	Write the phrase as one long line.
(the difference between a number and 4) + (the sum of the number and 8)	"added to" is replaced with addition.
$(x - 4) + (x + 8)$	Difference indicates subtraction. Sum indicates addition.
$2x + 4$	Collect like terms.

Sometimes one variable is needed to express two concepts. The second number can be figured out by subtracting. Consider the following sentence and a few random possible values for the first number: "The sum of two numbers is 20." That would lead to following scenarios.

First number	Second number	Operation
12	8	20 – 12 = 8
7	13	20 – 7 = 13
18	2	20 – 2 = 18
x	?	20 – x

That is the key. If the sum of two numbers is 20, let the first number be x and the second number is 20 – x.

Examples

4. Translate into a variable expression and simplify:

The sum of two numbers is fifteen. Using z to represent the larger number, translate "eighteen more than the smaller number".

Steps	Reasons
"18 more than the smaller number"	We need to translate.
the smaller number is $15 - z$	If z represents the larger number and the sum of two numbers is 15, then the smaller number is $15 - z$ as shown above
18 more than $15 - z$	Replace the smaller number with $15 - z$. More than means add.
$15 - z + 18$	
$33 - z$	Collect like terms to simplify.

5. Translate into a variable expression and simplify:

The sum of two numbers is thirty. Using p to represent the larger number, translate "the difference between twice the smaller number and seven" into a variable expression.

<u>Steps</u>	<u>Reasons</u>
the difference between twice the smaller number and 7	Translate.
the smaller number is $30 - p$	If p represents the larger number and the sum of two numbers is 30, then the smaller number is $30 - p$. See the explanation before this set of examples.
the difference between twice $(30 - p)$ and 7	Replace the smaller number with $30 - p$.
twice $(30 - p)$ – 7	Difference indicates subtraction. Replace "and" with subtraction.
$2(30 - p) - 7$	Twice means multiply by 2.
$60 - 2p - 7$	Simplify.
$-2p + 53$	Collect like terms.

Now that we have been translating into variable expressions, we will translate phrases into <u>variable equations</u>. Variable equations include an equality sign as well as numbers, variables, and arithmetic operations. Translating words into equations allows us to find an unknown value when we solve the equations.

Keys to “translating word” problems:

1. Let a variable stand for what you are asked to find, which will be “the number”.
2. Writing = where indicated from the beginning splits the more complicated sentence into two parts.
3. Translate the two parts of the problem.
4. Solve and state answer.

Examples

6. A number increased by twenty-seven is fifty. Find that number.

Steps	Reasons
x = the number	It is usually a good idea to let the variable stand for what you are looking for.
A number increased by twenty-seven is fifty. $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ x + 27 = 50	Write the sentence on a single line. Translate individual words.
$x + 27 = 50$ $-27 \quad -27$ $x = 23$	Solve the equation.

So, the number is 23.

7. Three hundred represents twice a number. Find that number.

Steps	Reasons
x = the number	It is a good idea to let the variable stand for what you are looking for.
Three hundred represents twice a number. $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ 300 = 2 · x	Write the sentence on a single line. Translate individual words. Look for the “= word”, which is “represents”.
$300 = 2x$ $\frac{300}{2} = \frac{2x}{2}$ $150 = x \text{ or } x = 150$	Solve the equation.

The number is 150.

8. Negative three-eighths is equal to the product of two-thirds and some number. Find the number.
Solution:
Write the phrase and translate the different parts. "is equal to" splits the problem in half. Product means we multiply.

Negative three-eighths equals the product of two-thirds and some number

$-\frac{3}{8} \quad = \quad \frac{2}{3} \quad \cdot \quad x$

$-\frac{3}{8}=\frac{2}{3}x$ Solve the resulting equation. The fraction is connected to the variable by multiplication. We should divide both sides by $\frac{2}{3}$.

$-\frac{3}{8}\cdot\frac{3}{2}=\frac{\cancel{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}x$ Dividing by $\frac{2}{3}$ is the same as multiplying by $\frac{3}{2}$.

$-\frac{9}{16}=x$ Do the multiplication.

$x=-\frac{9}{16}$ Write your answer with the variable on the left.

The number is $-\frac{9}{16}$. State the answer.

9. The difference between a number and negative seven is negative sixteen. Find the number.

Steps	Reasons
x = the number	Let a variable stand for the quantity that you are trying to find.
The difference between a number and –7 is –16. x – –7 = –16	Write out the sentence in a row. Translate the numbers immediately.

$x-(-7)=-16$
$x+7=-16$ Difference represents subtraction. Subtraction replaces the “and.“ = replaces “is”

$x+7=-16$
$\;\;-7 \quad -7$ Solve the equation: Simplify the – (–) as +

$x=-23$
The number is –23. Because the number is connected to the variable by addition, subtract both sides by 7.

10. The sum of two numbers is thirty-three. Twice the smaller is three more than the larger number. Find the two numbers.

Steps	Reasons
x = the smaller number? **= the larger number?**	I am looking for two numbers. One of them will be my variable. The other one also needs to be named, but that needs to wait. I do not want another variable because we do not know how to solve equations with more than one variable.
Sum of two numbers is 33. **Twice smaller is 3 more than larger.**	List both pieces of information.
Sum of two numbers is 33. **Smaller + larger = 33** **–smaller – smaller** **= 33 – x**	Look at the simpler piece of information. If they add up to 33, then we can solve for either number by subtracting on both sides. It would be great for students to know **that if the sum is 33 then the numbers being added are x and 33 – x.**
Twice smaller is 3 more than larger. **2(x) = (33 – x) + 3**	Now look at the more complicated piece of information. Substitute smaller with x and larger with 33-x. Twice means multiply by 2, 3 more than means add 3, and "is" means =.
$2x = 33 - x + 3$ $+x \quad\quad +x$ $3x = 36$ $\frac{3x}{3} = \frac{36}{3}$ $x = 12$	Solve the equation
x = 12 **33 –x = 33 – 12 = 21**	x = 12 so find the larger number by evaluating 33 – x for x = 12.
The numbers are 12 and 21.	State your answer.

Check your answers by looking at the original problem.
12 + 21 = 33
2(12) = 3 more than 21.

Exercises

Translate each of the following to an algebraic expression and simplify if possible. Remember that algebraic expressions do not include an equal sign.

1. The sum of five and three times a number

2. The difference between a number and fifty percent of the number

3. The difference between twice a number and 5

4. The sum of seven more than twice a number and the number

5. The difference between negative three times a number and five more than twice the number

6. The sum of twice a number decreased by ten and twenty percent of the number

7. Twenty less than twelve percent of a number minus the sum of twice the number and negative fifteen

8. Thirty more than eighty percent of a number added to the difference of three times the number and negative twenty-five

9. The sum of two numbers is thirty. Using z to represent the larger number, translate "twenty more than the smaller number" into a variable expression and simplify.

10. The sum of two numbers is forty. Using x to represent the larger number, translate "fifty more than the smaller number" into a variable expression and simplify.

11. The sum of two numbers is ten. Using x to represent the smaller number, translate "the difference between twice the larger number and negative twenty" into a variable expression and simplify.

12. The sum of two numbers is twenty-five. Using z to represent the smaller number, translate "the sum of twice the larger number and fifteen" into a variable expression and simplify.

Translate the following into an algebraic equation and solve to find the missing number. Remember that variable equations do have an equal sign.

13. The difference between a number and negative twenty-five is seventy-three. Find the number.

14. Three-fourths of a number minus five-eighths equals one-half. Find the number.

15. The sum of two-thirds of a number and negative one-ninth is five-sixths. Find the number.

16. The sum of two numbers is thirty-four. The sum of twice the smaller number and fifteen is thirty-nine. Find the two numbers.

17. The sum of two numbers is forty. The difference between twice the smaller and forty-two is negative fourteen. Find both numbers.

18. The sum of two numbers is fifteen. The difference between twice the larger number and twenty-two is eight. Find both numbers.

19. The sum of two numbers is negative seven. The sum of twice the smaller number and seventy is six. Find both numbers.

Application problems or word problems are often a source of frustration for many students. Not to worry! We can improve our problem solving skills. Frequently students are overwhelmed by the information and do not know where to begin or they protest that they do not know what is being asked. If there is a question mark, look there! In algebra, we often let a variable stand for what we are being asked. Following the steps below will help us to become better problem solvers.

General Strategy for Solving Application Problems

1. Familiarize yourself with the problem:
   - Read the problem perhaps several times.
   - Identify the question. Frequently, let a variable stand for what is being asked.
   - List information, write down important formulas, pictures
   - If you can use some information to let one variable stand for two unknowns, do it.
   - Charts: Many problems can be organized by using charts. There are specific types of charts for different types of problems.
2. Translate to a solvable problem:
   - We translate word problems into equations that can be solved.
3. Solve the equation.
   - Usually the equations are not too complicated.
4. Check:
   - Does the answer make sense? Remember, bicycles to do not go 300 mph, planes do not fly 4 miles in 7 hours, butterflies do not weigh 750 kilograms, etc.
5. State answer:
   - Use a sentence. Most word problems have a context and it is important to state whether we are talking about area or length or speed or time.
   - Use the correct units. Depending on the problem, the correct answer may use square feet, meters, miles per hour or seconds.
6. Look Back:
   - To learn how to do word problems it is important to apply the correct methods to different types of problems. After you struggle to finish a word problem you need to take some time to focus on what you did well to solve the problem.
   - Ask yourself two questions:
     1. What type of problem is being solved?
     2. What method is employed to solve the problem? Especially pay attention to the types of charts that are being used.

Do look back! It is extremely helpful when learning how to solve word problems.

Examples

1. Find three consecutive odd integers such that twice the difference of the third and first integers is fifteen less than the second integer.

Steps	Reasons
x = first odd integer x + 2 = second odd integer x + 4 = third odd integer	Use 1 variable to name all three integers.

twice the difference of the third and first integers is 15 less than the second integer

2 · (difference of the third and first integers) = second integer – 15

2 · [(x + 4) – x] = x + 2 – 15

$2[(x+4)-x]=(x+2)-15$	Solve for x.
$2[x+4-x]=x+2-15$ $2(4)=x-13$	The parentheses can be dropped because there is nothing in front of the parentheses to distribute. Collect like terms.
$8=x-13$	
$21=x$	Get x by itself by adding 13 to both sides.
$x=21$ $x+2=21+2=23$ $x+4=21+4=25$	Find the other odd integers by adding 2 and 4.
The three integers are 21, 23, and 25.	State answer.

2. A stamp collector bought 250 stamps for \$61.50. The purchase included 10¢ stamps, 25¢ stamps, and 30¢ stamps. The number of 25¢ stamps is five times the number of 10¢ stamps. How many 30¢ stamps were purchased?

number of 25¢ stamps is 5 times the number of 10¢ stamps.

number of 25¢ stamps = 5 · number of 10¢ stamps
number of 25¢ stamps = 5 · x
number of 10¢ stamps = x

We can use a chart to get the appropriate equation.

	Number of stamps	Cost of each stamp	Total cost
10¢ Stamps	x	.10	.10x
			+
25¢ Stamps	5x	.25	.25(5x)
			+
30¢ Stamps	250 – 6x ???	.30	.3(250 – 6x)
			=
Totals	250		61.50

Notice: We are looking for the number of 30¢ stamps, which is equal to the total number of stamps minus the number of 10¢ number and minus the number of 25¢ stamps. Add the cost for the 10¢, 25¢, and 30¢ stamps to get the cost of the collection.

Solve the equation from the Total cost column of the chart.

$$.10x + .25(5x) + .3(250 - 6x) = 61.5$$

Solve the equation.

Distribute the .3

$$.10x + 1.25x + 75 - 1.8x = 61.5$$

Collect like terms:
.10 + 1.25 – 1.8 = –.45

$$-.45x + 75 = 61.5$$

Subtract 132 from both sides.

$$-.45x = -13.5$$

Divide both sides by –.45

$$x = \frac{-13.5}{-.45}$$

$$x = 30$$

x = 30
250 – 6x = 250 – 6·(30) = 250 – 180 = 70 Get the number 30¢ stamps

The stamp collector bought seventy 30¢ stamps. State the answer.

3. An IQ training program claims that for every hour of training an individual will improve their IQ by 1.5. If we accept that claim as fact, how many hours of training will it take for somebody with an IQ of 84 to raise their IQ score to 150?

Let x stand for what we are trying to find:
x = the number of hours of training??? (the question)

Think of the process:

New IQ equals the old IQ plus increase from training
New IQ = 84 + 1.5 times the number of hours if training

150 = 84 + 1.5 x

Solve the equation:

$150 = 84 + 1.5\mathrm{x}$

$150 - 84 = 1.5\mathrm{x}$

$66 = 1.5\mathrm{x}$

$\frac{66}{1.5} = \mathrm{x}$

$\mathrm{x} = 44$

If the claim were to be true after 44 hours of training, somebody could improve their IQ from 84 to 150.

4. The average weight for three year old girls is 30 pounds. If each year the average female child gains 7.2 pounds, how old will the average girl be when she weighs 102 pounds?

<u>List information:</u>
3 years old ⟶ 30 pounds
Each year after 3 years old ⟶ 7.2 pounds

Since 3 years old is the starting point and we are looking for the age or number of years old we will let:
x = the number of years after 3 years old

End weight = starting weight + weight added each year
102 = 30 + 7.2 times number of years after 3 (which is x)

$102 = 30 + 7.2x$ Write the equation.

$102 - 30 = 7.2x$

$72 = 7.2x$ Solve the equation.

$\frac{72}{7.2} = x$

$x = 10$

The average girl will weigh 102 pounds when she is 13 years old.

$10 + 3 = 13$

5. The local swimming pool has two summer plans to choose from. For the first plan an individual pays \$100 for the whole summer with unlimited use. For the second plan individuals pay \$50 for the summer and then \$2.50 per day that they use the pool. After how many days will the costs of the two plans be the same?

When comparing two different situations a chart with a line down the middle helps organize the information.

First plan	Second plan
\$100 for the summer	\$50 for the summer \$2.50 per day used

Total cost 100 | Same Cost | Total cost 50 + 2.50 x (number of days using the pool)

x= the number of days using the pool?

$100 = 50 + 2.50\text{x}$ Write the equation.
$100 - 50 = 2.50x$
$50 = 2.50x$ Solve the equation.
$\frac{50}{2.50} = x$ After 20 days of using the pool the cost of both plans is the same.
$20 = x$

6. Janet wants to know which cell phone plan will suit her best. The first plan costs \$35 per month and \$0.10 for each additional minute after the first 90 minutes. The second plan costs \$30 per month with a cost of \$0.15 after the first 90 minutes. After how many minutes will the cost of the two plans be the same?

When comparing two different situations a chart with a line down the middle helps organize the information.

First plan	Second plan
\$35 per month \$0.10 per minute after 90 minutes	\$30 per month \$0.15 per minute after 90 minutes
Total Cost 35 + 0.10 x (number of minutes more than 90)	Total Cost 30 + 0.15 x (number of minutes more than 90)

Same Cost

x= the number of minutes after the first 90 minutes??
Use the equality sign for the same cost.
$35 + 0.10\text{x} = 30 + 0.15\text{x}$ Write the equation.
$35 - 30 = 0.15\text{x} - 0.10\text{x}$ Solve.
$5 = 0.05\text{x}$
$\frac{5}{.05} = \text{x}$
$\text{x} = 100$
After 100 + 90 or 190 minutes of calls the cost of the two cell phone plans will be the same.

Looking back we really have a few different types of problems. Example 1 involves consecutive odd integers. Odd integers are two apart (1,3,5,7,…). So, we can label the unknown integers x, x+2, x+4. Consecutive even integers (2,4,6,8,…) would work the same way since they are also two apart. Consecutive integers (1,2,3,4,…) are one apart. If a problems asks about consecutive integers, they can be labeled x, x+1, x+2.

Example 2 is a stamp problem. It is much like a translating problem to get the different names of the unknown quantities of stamps. Multiplying the number of stamps by the value of the stamps lets us get the total value for each type of stamp, which can be added together to get the total value of the collection.

Examples 3 and 4 start at a given level and then increase for each hour of training or year of growth. So, the end value is equal to the beginning values plus the hourly or yearly increase times the number of hours or years.

In examples 5 and 6, we are comparing two situations. When comparing two situations, it is useful to make a chart to organize the different information. The two types of information are connected in some way. In the above examples, we are looking for the condition that will make the two costs the same.

Exercises

To solve the following word problems use algebraic equations. State your answer with the correct units if appropriate.

1. Find three consecutive odd integers such that the three times the difference of the third and first odd integer is five less than the second odd integer.

2. Find three consecutive even integers such that twice the sum of the first and third even integers is forty-four more than twice the second even integer.

3. Find three consecutive integers such that five times the third integer is eleven less than three times the sum of the first and the second integers.

4. Find three consecutive integers such that six times the first integer is sixteen more than the twice the sum of the second and third integers.

5. A stamp collector bought 160 stamps for $25.00. The purchase included 5¢ stamps, 15¢ stamps, and 25¢ stamps. The number of 15¢ stamps is three times the number of 5¢ stamps. How many of each type of stamp was purchased?

6. A stamp collector bought 295 stamps for $87.75. The purchase included 5¢ stamps, 25¢ stamps, and 40¢ stamps. The number of 40¢ stamps is fifty more than twice the number of 5¢ stamps. How many of each type of stamp was purchased?

7. If the average salary for a college instructor in 1925 was $2300 and that average salary increased each year by $180, in what year was the average salary for a college instructor $10,040?

8. If the average salary for a young businessperson in 1943 was $4530 and that salary increased by $370 each year, in what year was the average salary for a young businessperson $17,110?

9. A printer that costs $60 requires replacement cartridges that sell for $23. Over the life of the printer the cost of the printer and replacement cartridges is $865. How many cartridges were used over the life of the printer?

10. The air pressure at sea level is about 14.7 pounds per square inch (psi). For every foot above sea level, the air pressure drops about 0.0005263 psi. According to this information, at about what height above sea level is the air pressure 13.2 psi?

11. The Scholastic Aptitude Test (SAT) is used by universities to help determine if a candidate should be allowed to enter the school. One year at Stanford 54% of the students that were admitted had a SAT Math score of 700 or better. An SAT training program claims that for every hour of training an individual will improve their SAT Math score by 3 points. If we accept that claim as fact, how many hours of training will it take for somebody with a SAT Math score of 529 to raise their SAT Math score to 700?

12. The average weight for four year old boys is 34.5 pounds. If each year the average boy gains 8.2 pounds, how old will the average boy be when he weighs 124.7 pounds?

13. A day care offers two options. Plan 1 allows a child to attend the day care any time during the work week for a cost of $275 per month. Plan 2 requires a $50 per month fee plus $3.75 per hour. After how many hours will the costs of the two plans be the same?

14. An electric company will allow its clients to pick one of two different options. For the first option clients pay a $37.24 basic services fee plus $0.096 per kilowatt hour. The second option allows clients to pay a $24.36 basic services fee with a cost of $0.124 per kilowatt hour. After how many kilowatt hours will the costs of the two plans be the same?

15. An electric company will allow its clients to pick one of two different options. For the first option clients pay a $27.25 basic services fee plus $0.162 per kilowatt hour. The second option allows clients to pay a $43.49 basic services fee with a cost of $0.104 per kilowatt hour. After how many kilowatt hours will the costs of the two plans be the same?

16. Rental car company A charges $125 per week with a $0.04 charge per mile and rental car company B charges $59 per week with a $0.10 charge per mile. After how many miles with the cost of the two rental car companies be the same?

Ratios are quotients or quantities with the same units. The quotient is the result of division. Division can be written as a fraction.

Words	Symbols	Meaning
2 cups of sugar to 8 cups of milk	$2:8 \ or \ 2 \div 8 \ or \ \frac{2}{8}$	$\frac{2 \ cups \ of \ sugar}{8 \ cups \ of \ milk}$

These three ways of expressing a ratio all have the same meaning.

Simplify ratios by writing them as fractions.

Example

1. Simplify: 2 cups of sugar to 8 cups of milk

Steps	Reasons
2 cups of sugar to 8 cups of milk	Translate to fraction notation.
$\frac{2}{8} = \frac{1}{4}$	Simplify the fraction.
$\frac{1}{4} \ or \ 1:4 \ or \ 1 \ to \ 4$	You can write your answer three ways.

Rates are quotients of quantities with different units. The quotient is the result of division. Division can be written as a fraction.

Words	Symbols	Meaning
192 dollars to 150 euros	192 dollars : 150 euros	$\frac{192 \ dollars}{150 \ euros}$

Simplify rates by writing them as fractions.

Example

1. Simplify: 192 dollars to 150 euro

Steps	Reasons
192 dollars to 150 euro	Translate to fraction notation.
$\frac{192}{150} = \frac{6 \cdot 32}{6 \cdot 25} = \frac{32}{25}$	Simplify the fraction.
$\frac{32 \ dollars}{25 \ euros} \ or \ 32 \ dollars : 25 \ euros \ or \ \$32 \ to \ 25 \ euros$	You can write your answer three ways.

A unit rate has a 1 in the denominator. To get a unit rate just divide with a calculator. Unit rates can be used to convert units.

Example

2. Write 192 dollars to 150 euro as a unit rate.

Steps	Reasons
192 dollars to 150 euro	Translate to fraction notation.
$\frac{192}{150} = 1.28$	Divide.
\$1.28 / 1 euro	Write the answer with units.

\$1.28 / euro also can be written \$1.28 : 1 euro. We can use this unit fraction when making conversions euro to dollar.

Proportions are an equality of ratios, rates, or fractions.

To check if a proportion is true or to solve for a variable we can "cross multiply."

We know $\frac{2}{3} = \frac{6}{9}$.

We can check by "cross multiplying" $\frac{2}{3} \times \frac{6}{9}$ $\quad 2 \cdot 9 = 18$ and $3 \cdot 6 = 18$

Since the fractions are equal, we do get the same number when we "cross multiply."

We know $\frac{1}{2} \neq \frac{3}{4}$

We can check by "cross multiplying" $\frac{1}{2} \times \frac{3}{4}$ $\quad 1 \cdot 4 = 4$ and $2 \cdot 3 = 6$

Since the fractions are not equal, we do not get the same number when we "cross multiply."

To solve proportions:
1. "cross multiply"
2. Solve the resulting equations.

Examples

3. Solve: $\frac{x}{25} = \frac{7}{5}$

Steps	Reasons
$\frac{x}{25} = \frac{7}{5}$	We want to solve a proportion.
$\frac{x}{25} \quad \frac{7}{5}$	Cross Multiply
$5x = 25 \times 7$	
$5x = 175$	Solve the equation.
$\frac{5x}{5} = \frac{175}{5}$	
$x = 35$	

Check the equation:

$\frac{35}{25} = \frac{7}{5}$ is true. When $\frac{35}{25}$ is simplified, it is $\frac{7}{5}$.

4. Solve: $\frac{3}{8} = \frac{9}{x+3}$

Steps	Reasons
$\frac{3}{8} = \frac{9}{x+3}$	We want to solve a proportion.
$\frac{3}{8} \quad \frac{9}{x+3}$	Cross Multiply Use parentheses to multiply 3 and x+3
$3(x+3) = 8 \cdot 9$	Solve the equation:

$$3x + 9 = 72$$
$$-9 \quad -9$$
$$3x = 63$$
$$\frac{3x}{3} = \frac{63}{3}$$
$$x = 21$$

Short-cut:

$3x + 9 = 72$	Notice that we can subtract 9 from both sides which gets rid of the 9 on the left.
$3x = 63$	Dividing by 3 on both sides gets rid of the 3 on the left.
$x = 21$	

We save time using this short-cut method.

5. A car is driven 162 miles and uses 7.5 gallons of gasoline. At this rate how far will the car go with 2.5 gallons of gasoline?

<u>Steps</u>	<u>Reasons</u>
162 miles : 7.5 gallons x = miles?? : 2.5 gallons	Write both rates (or ratios). Keep the same units in the same column. Let a variable stand for the missing piece of information.
$\frac{162}{7.5} = \frac{x}{2.5}$	Write the proportion: 162 miles is to 7.5 gallons as what number of miles is to 2.5 gallons.
$\frac{162}{7.5} \quad \frac{x}{2.5}$	Cross Multiply
$7.5x = 162 \cdot 2.5$	
$7.5x = 162 \cdot 2.5$	Solve the equation.
$7.5x = 405$	
$\frac{7.5x}{7.5} = \frac{405}{7.5}$	
$x = 54$	
The car would go 54 miles with 2.5 gallons of gas.	Write a sentence to state the answer with the correct units.

6. In a wilderness area, 8 bears are caught, tagged, and released. Later 20 bears are caught and 5 have tags. Estimate the number of bears in the wilderness area.

<u>Steps</u>	<u>Reasons</u>
8 bear tagged : x = all bear in area 5 tagged : 20 captured	Try to set up two ratios:
So, we get the proportion $\frac{8}{x} = \frac{5}{20}$	The trick is to see that there are a total of 8 bears tagged in the whole area.
$\frac{8}{x} = \frac{5}{20}$	Cross multiply to solve proportions.
$8 \cdot 20 = 5x$ $160 = 5x$ $x = 32$	There are approximately 32 bears in the wilderness area

7. After working for a company for 15 months, an employee has earned 20 vacation days. At this rate, how much longer will the employee have to work in order to get a 6-week vacation (30 work days)?

Steps	Reasons
15 months : 20 vacat. days 15 +x months :30 vacat. days	Write the proportion as months to vacation days. Let x=number of months in addition to the 15
$\frac{15}{20} = \frac{15+x}{30}$	Write the proportion and cross-multiply to solve.
$15 \cdot 30 = 20(15+x)$	Solve the resulting equation by distributing the 20.
$450 = 300 + 20x$	
$150 = 20x$	
$\frac{150}{20} = x$	
$x = 7.5$	

The employee needs to work 7.5 months more to get the 6-week vacation.

8. An investment of $3500 earns $700 each year. At the same rate, how much additional money must be invested to earn $1200 each year?

Steps	Reasons
investment $3500 : earns $700 x + 3500 : earns $1200 x=additional amount to invest??	Write both rates (or ratios). Keep the same type of information in the same column. Let a variable (x) stand for the missing piece of information. Here we want the additional amount more than the $3500 to invest.
$\frac{3500}{700} = \frac{x+3500}{1200}$	Write the proportion.
$\frac{3500}{700} = \frac{x+3500}{1200}$	Cross-multiply.

$700(x + 3500) = 3500 \cdot 1200$

Solve the equation.

$$700x + 2{,}450{,}000 = 4{,}200{,}000$$
$$-2{,}450{,}000 \quad -2{,}450{,}000$$

$$700x = 1{,}750{,}000$$

$$\frac{700x}{700} = \frac{1{,}750{,}000}{700}$$

$$x = 2500$$

Short-cut:

$700x + 2{,}450{,}000 = 4{,}200{,}000$ Subtract 2,450,00 from both sides.

$700x = 1{,}750{,}000$ Divide both sides by 700.

$x = 2500$

The above short-cut is quicker than what we did on the left. Either way is fine.

$2500 more must be invested to earn $1200 each year.

Exercises

Express the following as unit rates:

1. 220 EUR (European Euro) is paid for an item costing 275 USD (US Dollars)

2. 56.70 USD (US Dollars) is paid for an item costing 35 GBP (Great Britain Pound)

3. 183.52 CAD (Canadian dollar) is paid for an item costing 148 USD (US Dollars)

Solve the following proportions:

4. $\frac{10}{6} = \frac{25}{x}$

5. $\frac{6}{21} = \frac{x}{56}$

6. $\frac{36}{x} = \frac{12}{3}$

7. $\frac{x}{72} = \frac{5}{12}$

8. $\frac{x+2}{12} = \frac{15}{18}$

9. $\frac{6}{x-3} = \frac{8}{12}$

10. $\frac{14}{3} = \frac{35}{x-4}$

11. $\frac{x-5}{6} = \frac{x+4}{9}$

12. $\frac{9}{x-3} = \frac{15}{x+3}$

13. A car is driven 127.4 miles and uses 5.2 gallons of gasoline. At this rate how far will the car go with 3.6 gallons of gasoline?

14. A car is driven 317.1 km and uses 30.2 liters of gasoline. At this rate how far will the car go with 40.5 liters of gasoline?

15. In order to estimate the number of elephants in an enclosed habitat, 28 elephants are caught, tagged, and released. Later 105 elephants are observed and 12 have the tags. Estimate the number of elephants in the enclosed habitat.

16. In a nature preserve 126 elk are caught and tagged. Later 75 elk are caught in the same preserve and 45 are found to have tags. Estimate the number of elk in the nature preserve.

17. In a wildlife refuge 42 bears are caught and tagged. Later 21 bears are caught in the same wildlife refuge and 14 are found to have tags. Estimate the number of bears in the wildlife refuge?

18. After working for a company for 12 months, an employee has earned 15 vacation days. At this rate, how much longer will the employee have to work in order to get a 4-week vacation (20 work days)?

19. After working for a company for 18 months, an employee accrued 12 vacation days. At this rate, how much longer will the employee have to work in order to get a 5-week vacation (25 work days)?

20. An investment of $6200 earns $800 each year. At the same rate, how much additional money must be invested to earn $1500 each year?

21. An investment of $4500 earns $300 each year. At the same rate, how much additional money must be invested to earn $450 each year?

Percent means per 100. We can replace the percent symbol (%) with multiplication by $\frac{1}{100}$ or 0.01.

Examples

(Changing percents to decimals.)

1. Write 37.5% as a decimal.

Steps	Reasons
37.5%	
	Replace % with multiplication by 0.01.
$37.5 \cdot 0.01$	
	Multiplying by 0.01 moves the decimal point to places to the left.
$.375$	

(Changing percents to fractions.)

2. Write 25% as a fraction.

Steps	Reasons
$25\% = 25 \cdot \frac{1}{100}$	Replace % with $\frac{1}{100}$.
$\frac{25}{1} \cdot \frac{1}{100} = \frac{25}{100}$ $= \frac{1}{4}$	Write the whole number as an improper fraction, multiply, and simplify the fractions.

3. Write $15\frac{2}{5}\%$ as a fraction.

Steps	Reasons
$15\frac{2}{5} \cdot \frac{1}{100}$	Replace % with $\frac{1}{100}$.
$\frac{77}{5} \cdot \frac{1}{100} = \frac{77}{500}$	Write the mixed number as an improper fraction and multiply the fractions.

100% is equal to one. When changing decimal numbers or fractions to percents, we can just multiply by 100% because we are multiplying by one.

(Changing fractions to percents.)

4. Write $\frac{9}{20}$ as a percent.

Steps	Reasons
$\frac{9}{20}$	Multiply by 100%, which is 1.
$\frac{9}{20} \cdot 100\%$	
$\frac{9}{20} \cdot \frac{100}{1}\%$	Write 100 as an improper fraction, multiply, and simplify the fractions.
$\frac{9}{20} \cdot \frac{5 \cdot 20}{1}\%$	
$\frac{45}{1}\%$	
45%	

5. Write $\frac{5}{6}$ as a percent.

Steps	Reasons
$\frac{5}{6} \cdot 100\%$	Multiply by 100%.
$\frac{5}{6} \cdot \frac{100}{1}\%$	Write 100 as an improper fraction, multiply, and simplify the fractions. Write your answer using a mixed number if necessary.
$\frac{5}{2 \cdot 3} \cdot \frac{2 \cdot 50}{1}\%$	
$\frac{250}{3}\%$	
$83\frac{1}{3}\%$	

(Changing decimals to percents.)

6. Write 0.64 as a percent.

Steps	Reasons
0.64	
	Multiply by 100%.
$0.64 \cdot 100\%$	Multiplying by 100% moves the decimal point to places to the right.
64%	

7. Write 14.2 as a percent.

Steps	Reasons
14.2	
	Multiply by 100%.
$14.2 \cdot 100\%$	Multiplying by 100% moves the decimal point to places to the right.
1420%	

It may seem strange to get 1420%, but it makes more sense if we keep in mind that 1=100%. So a number more than 1 has a value of more than 100%.

The basic percent equation states that the Amount is the Percent of the Base or A = P · B. It may be easier to translate directly using = for "is" and multiply by percent for "percent of".

Examples

8. 35% of 80 is 28.

$(.35) \cdot 80 = 28$

Translate "is" to "=" .
Translate "35%" to .35
Translate "of" to multiplication
Translate the other numbers directly.

One way to solve these percent problems is to translate directly to an equation.

Examples

(Basic percent equation.)

9. 20 is 40% of what? Use the basic percent equation.

Steps	Reasons
20 is 40% of what? 20 = .40 · x	Translate "is" to "=" . Let a variable stand for what you are looking for. Translate "of" to multiplication Change 40% to .40 and write the numbers.
$20 = .4x$	Solve the equation.
$\frac{20}{.4} = \frac{.4x}{.4}$	
$50 = x$	

20 is 40% of 50.

10. 120 is what percent of 800? Use the basic percent equation.

Steps	Reasons
120 is what percent of 800? 120 = P · 800	Translate "is" to "=" . Let a variable stand for what you are looking for. Translate "of" to multiplication
$120 = P \cdot 800$	Solve the equation.
$\frac{120}{800} = \frac{P \cdot 800}{800}$	
$.15 = P$	
$P = 15\%$	Change the decimal number to a percent. $.15 = .15 \cdot 100\% = 15\%$

120 is 15% of 800.

To solve percent increase and percent decrease problems we use three pieces of information.

1. **Base:** first value before the increase or decrease. We could think of this as the original amount before the increase or decrease.

2. **Amount:** amount of the increase or decrease;
   - final value after increase minus the first value before the increase
   - first value before decrease minus final value after the decrease

3. **Percent:** percent increase or percent decrease

Examples (Percent increases)

11. There were 8.2 million people in a large city in the year 1990. In the year 2000, there were 10.1 million people living there. Find the percent increase in the city's population from 1990 to 2000.

Steps	Reasons
Base: 8.2 million Amount: 10.1-8.2 = 1.9 mill. Percent: ???	1. **Base:** first value before the increase (original amount) 2. **Amount:** amount of the increase; final value after increase minus first value before increase 3. **Percent:** percent increase is what we are asked to find.
$P \cdot B = A$ $P \cdot (8.2) = 1.9$ $\frac{P \cdot (8.2)}{8.2} = \frac{1.9}{8.2}$	Here we are saying that the percent of the increase times the original amount is equal to the amount of the increase.
$P = .2317 \ \ or \ \ 23.2\%$	Change the decimal to a percent.
There was a 23.2% increase in the city's population from 1990 to 2000.	State your answer.

Example (Percent decreases)

12. It is estimated that the value of a new car drops 45% in the first two years. Find the value of a $24,900 new car after two years.

Steps	Reasons
Base: 24,900 Amount: 24,900 – unknown = ???? Percent: 45%	1. **Base:** first value before the decrease 2. **Amount:** amount of the decrease; first value before decrease minus final value after decrease. Here we are looking for the final value after the decrease. 3. **Percent:** percent decrease
$P \cdot B = A$	Here we are saying that the percent of the decrease times the original amount is equal to the amount of the decrease.
$.45 \cdot 24{,}900 = A$	We get the amount of the decrease.
$11{,}205 = A$	
original value – decrease = final value 24,900 – 11,205 = 13,695	We subtract the amount decreased from the new car price to get the price after two years.
The value of the car after two years is $13,695.	State the answer using a sentence and correct units.

There are a lot of equations that can be used to do mark-up (increases) and discount (decreases) equations. I will focus on the process, which is familiar to us. **Mark-up** is what a store does to make a profit. Sometimes stores sell their items at a **discount** so that they can sell them quickly.

Mark-up:

Cost of the item + Mark-up = Selling price

Mark-up = a percent of the cost.
(percent · cost)

It should make sense that a company has its cost for an item and then the company charges more in order to make a profit.

Examples: (Mark-up)

13. A shirt with a cost of $15 is sold for $26.70. Find the mark-up rate.

Steps	Reasons
Cost of the item + Mark-up = Selling price	• Write the basic concept behind mark-up rate. • Find the mark-up
15 + Mark-up = 26.7 –15 –15 Mark-up = 11.7	• Mark-up = a percent of the cost. • "Percent of" means multiply by a percent. So,
Mark-up = P · cost 11.70 = P (15)	• Mark-up = P · cost

$$11.7 = P(15)$$

Solve the equation.

$$\frac{11.7}{15} = \frac{P(15)}{15}$$

$$P = .78 \;\; or \;\; 78\%$$

There is a 78% mark-up.

14. A mark-up rate of 42% is applied to a tennis racquet costing $75. Find the selling price.

Steps	Reasons
Cost of the item + Mark-up = Selling price	• Write the basic concept behind mark-up.
75 + Mark-up = x	• Let x = selling price, because we are asked to find selling price.
75 + .42(75) = x	• Mark-up = a percent of the cost. • Mark-up = 42% of 75 or .42(75)

$$75 + .42(75) = x$$
$$75 + 31.5 = x$$
$$106.5 = x$$

Solve the equation.

The selling price is $106.50

Discounts:

Sale price = Regular price – Discount

Discount = percent of the regular price
(percent · regular price)

This way of thinking should make some sense. When an item is 20% off, we take a discount off the regular price and we say that discount is 20% of the regular price. It works just like a decrease problem where we take a percent of the original amount, which is called the regular price for an item on sale.

Examples:

15. A home entertainment system that has a regular price of $475 is on sale for 25% off the regular price. What is the sale price for the home entertainment system?

Steps	Reasons
Sale price = Regular price – Discount x = 475 – .25(475)	• Write the basic concept behind discount. • Discount is a percent of regular price. (25% of 475 = .25 · 475) • Let x = sale price, because we are asked to find sale price.
$x = 475 - .25(475)$ $x = 475 - 118.75$ $x = 356.25$	Solve the equation.
The sale price of the home entertainment system is $356.25	State your answer as a short sentence with the correct units.

16. A blender with a sale price of $48.75 is on sale for 35% off the regular price. Find the regular price.

Steps	Reasons
Sale price = Regular price – Discount 48.75 = x – .35 x	• Write the basic concept of a discount. • Let x = regular price, because we are asked to find regular price. • Discount is a percent of regular price. (35% of unknown = .35 · x). The trick is that we are taking 35% of what we are trying to find which is the regular price.
$48.75 = x - .35x$	Solve the equation.
$48.75 = 1x - .35x$ $48.75 = .65x$	Think of x as 1x. Then subtract 1 – .35 to collect like terms.
$\frac{48.75}{.65} = \frac{.65x}{.65}$	
$75 = x$	
The blender's regular price is $75.	State your answer.

Exercises

Change the percent to a decimal number:

1. 12%
2. 120%
3. 0.35%
4. 11.3%

Change the percent to a fraction:

5. 45%
6. $12\frac{3}{4}\%$
7. $47\frac{2}{3}\%$
8. 158%

Change the fraction to a percent:

9. $\frac{7}{25}$
10. $\frac{11}{40}$
11. $\frac{5}{9}$
12. $\frac{2}{15}$
13. $3\frac{1}{3}$
14. $2\frac{5}{12}$

Change the decimals to a percent:

15. 0.73

16. 0.148

17. 0.1682

18. 0.9453

19. 12.29

20. 125.42

Solve the following using the basic percent equation or by translating:

21. What is 27% of 160?

22. 42 is 35% of what?

23. 15.3 is 85% of what?

24. What is 60% of 120?

25. 312 is what percent of 600?

26. 54 is what percent of 45?

27. What is 240% of 80?

28. 72 is 150% of what?

29. 51 is 68% of what?

30. What is 320% of 5?

31. 153.6 is what percent of 240?

32. 486 is what percent of 300?

Solve the following:

33. Student enrollments at a university increased from 8900 to 10,502. Find the percent increase of the student enrollments at the university.

34. The number of city employees increased from 12,250 to 16,170 under a new mayor. What is the percent increase of city employees under the new mayor?

35. The number of jobs in a certain state decreased from 6.0 million to 5.1 million because of an economic crisis. What is the percent decrease in the number of jobs in the state because of the economic crisis?

36. Over his lifetime an adult male's height went from 73 inches to 71 inches. Find the percent decrease in height of the adult male to the nearest hundredth of a percent.

37. A blender that usually sells for $75 is on sale for 35% off of the original amount. What is the sale price of the blender?

38. A laptop that is normally $575 is sold at a discount of 15% off of the regular price. What is the sale price of the laptop?

39. A coffeemaker with a sale price of $95.04 is 28% off of the regular price. What is the regular price of the coffee maker?

40. A new car with a sale price of $22,360.80 is 16% off of the regular price. What is the regular price of the new car?

41. It is estimated that the value of a new car drops 24% in the first year. Find the value of a $32,900 new car after the first year.

42. A retailer buys a television for $380 and then marks up the price 45%. What is the retailers selling price for the television?

43. A wine shop buys a French Chardonnay for $6.50 and then marks up the price 80%. What is the wine shop's sale price for the French Chardonnay?

44. A brand name shirt has a cost of $24 for a small store. The store sells the shirt for $43.20 Find the mark-up rate.

45. A supermarket buys shrimp at a cost of $3.80 per pound. The supermarket sells the shrimp for $6.27 per pound. Find the mark-up rate.

Sets are denoted with braces { } and can be written two ways:

1. Roster Method: Lists the elements of the set.
2. Set-builder notation: we describe the set because it is too big to list all the elements.

Roster Method

A = {1,2,3,4,5} The set A has elements 1,2,3,4,5

B = {1,3,5,7,9 } The set B has elements 1,3,5,7,9.

The union of two sets is the set of all elements in either set or both sets. Union is denoted by $\cup$.

Find $A \cup B$ using A and B from above.

$$A \cup B = \{1,2,3,4,5,7,9\}$$

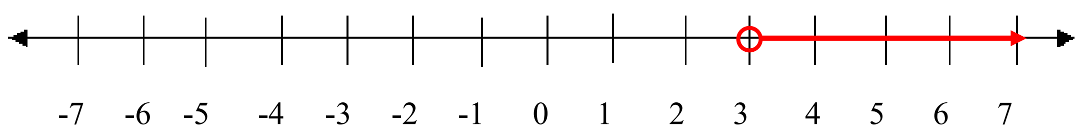
The intersection of two sets is the set of only the elements that are in both sets. Intersection is denoted by $\cap$.

Find $A \cap B$ using A and B from above.

$$A \cap B = \{1,3,5\}$$

Some sets are too big to list the elements. But we can graph them or write them in set-builder notation.

Consider the graph of x > 3



We use an open circle to show that the value 3 is not included.

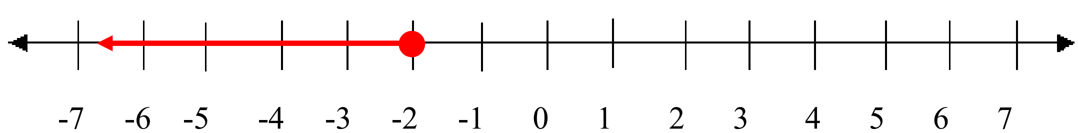
In set-builder notation we write

$$\{x \mid x > 3\}$$

Translated as: "the set of x such that x is greater than 3"

The braces indicate it is a set and the vertical line means "such that".

Consider the graph of $x \leq -2$



We use a closed circle to show that the value -2 is included.

In set-builder notation we write

$$\{x \mid x \leq -2\}$$

Translated as: "the set of x such that x is less than or equal to -2"

The braces indicate it is a set and the vertical line means "such that".

The difference between solving equations and solving inequalities is that we switch the inequality when we multiply or divide both sides by a negative number.

Think of the following example with numbers:

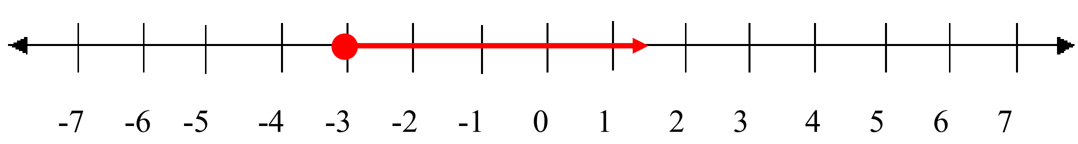
$-3 < 4$	We know that –3 is less than 4.
$-3(-2) > 4(-2)$	Multiply be a negative number switches the sign.
$6 > -8$	The result makes sense.

Examples:

1. Solve: $-3x \leq 9$

Steps	Reasons
$-3x \leq 9$	
$\frac{-3x}{-3} \geq \frac{9}{-3}$	Dividing both sides by a negative number switches the inequality.
$x \geq -3$	
$\{x \mid x \geq -3\}$	The answer can be written in set-builder notation.

The graph of $x \geq -3$

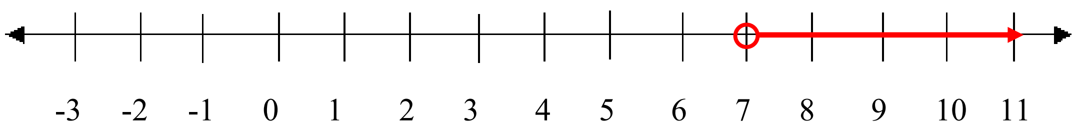


As with equations, we can solve more complicated inequalities by getting all the variables on one side and the numbers on the other. We may need to first simplify both sides of the inequalities.

2. Solve: $5x - 4 > 4x + 3$

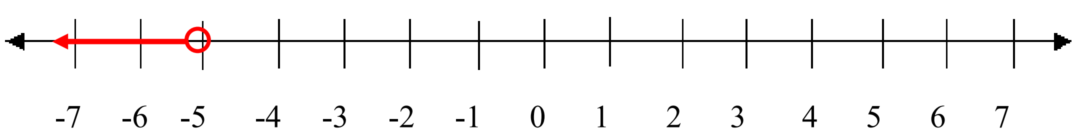
Steps	Reasons
$5x - 4 > 4x + 3$	
$-4x \quad -4x$	Subtracting the same number from both sides does not affect the inequality.
$x - 4 > 3$	Adding the same number to both sides does not affect the inequality.
$+4 \quad +4$	
$x > 7$	All done.
$\{x \mid x > 7\}$	The answer can be written in set-builder notation.

The graph of x > 7. There is an open circle at 7 because 7 is not included for >.



3. Solve: $2x - 11 > 5x + 4$

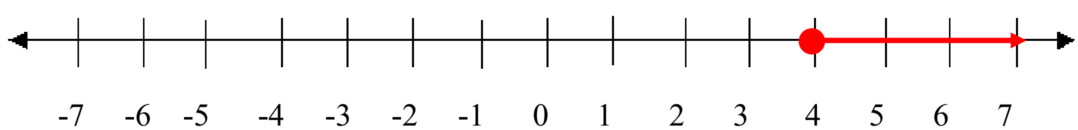
Steps	Reasons
$2x - 11 > 5x + 4$	
$2x - 5x > 4 + 11$	Add 11 and subtract 5x from both sides of the equation.
$-3x > 15$	Dividing both sides by a negative number switches the inequality.
$\frac{-3x}{-3} < \frac{15}{-3}$	
$x < -5$	
$\{x \mid x < -5\}$	We can write the answer using set-builder notation.



4. Solve: $3(2x-9)+2 \geq 17-2(x+5)$

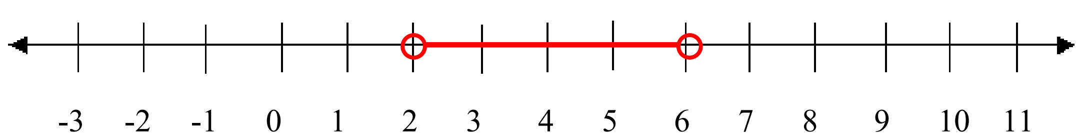
Steps	Reasons
$3(2x-9)+2 \geq 17-2(x+5)$	
$6x-27+2 \geq 17-2x-10$	Simplify both sides of the inequality.
$6x-25 \geq 7-2x$	Add 25 and add 2x to both sides of the inequality.
$6x+2x \geq 7+25$	
$8x \geq 32$	
$\frac{8x}{8} \geq \frac{32}{8}$	Dividing both sides by a positive number does not switch the inequality.
$x \geq 4$	
$\{x \mid \geq 4\}$	We can write the answer using set-builder notation.

We can also graph the inequality:



We use a closed circle to show that the 4 is included.

Sometimes we want to show that a number lies between two values. Consider the following graph:



Because of the open circles 2 and 6 are not included, but all of the values in between are included. There are few ways to express the inequality.

1. $2 < x < 6$
2. $x > 2$ and $x < 6$

We will use the first notation.

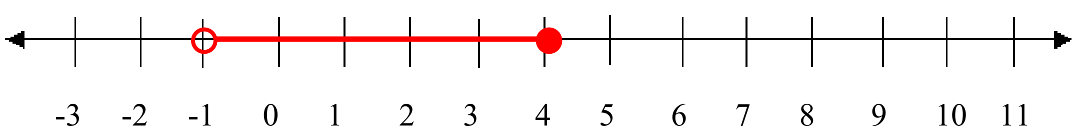
To solve these inequalities we will split the inequality into three parts (which are separated by the inequality) and do the same operations to all three parts at the same time.

Example:

5. Solve $-9 < 4x - 5 \leq 11$ write the solution in set notation and graph:

Steps	Reasons
$-9 < 4x - 5 \leq 11$ $+5 \quad\quad +5 \quad +5$	Split the inequality into three parts and try to get x alone in the middle: $-9 < 4x - 5 \leq 11$
$-4 < \quad 4x \quad \leq 16$	
$\frac{-4}{4} < \frac{4x}{4} \leq \frac{16}{4}$	Get x alone in the middle by first adding 5 then dividing by 4 from each of the three parts of the inequality.
$-1 < x \leq 4$	
$\{x \mid -1 < x \leq 4\}$	Write the answer in set notation.

Graph of the solution:



Examples:

6. In a math class there are four equally weighted tests. A student has grades of 92, 78, and 97 on the first three tests. To earn an A in the class, the students must have an average of at least 90. What must the student get on the fourth test to earn an A in the math class?

Steps	Reasons
Let x = grade on the fourth test.	Use a variable to stand for what we want to find. The question mark indicates what we are looking for.
Average at least 90 Average $\geq$ 90	Start by writing the words then translate the parts. At least means $\geq$
$\frac{92+78+97+x}{4} \geq 90$	To find the average, add the four grades and divide by four.
$\frac{4}{1} \cdot \frac{92+78+97+x}{4} \geq 90 \cdot 4$	To solve first multiply both sides by 4 to get rid of the fractions.
$92+78+97+x \geq 360$	
$267+x \geq 360$	Do the arithmetic and then subtract both sides by 267.
$x \geq 360-267$ $x \geq 93$	The student needs at least a 93 on the fourth test to earn a 90 for math class.

7. For a product to be labeled as orange juice a state agency requires that at least 65% of the drink is real orange juice. How many ounces of artificial flavors can be added to 26 ounces of real orange juice and have it still be legal to label the drink orange juice?

Steps	Reasons
x = amount of artificial flavors that may be added	Let a variable stand for what is being asked.
Amount of real juice must be at least 65% of total 26 ounces of real juice $\geq$ .65 (x+26) $26 \geq 0.65(x+26)$	At least means more than or equal. The total is the 26 ounces of real orange plus the artificial flavors that are added (x).
$26 \geq 0.65x+16.9$	
$9.1 \geq 0.65x$	Solve the inequality.
$\frac{9.1}{0.65} \geq \frac{0.65x}{0.65}$	

$14 \geq x \text{ or } x \leq 14$ No more than 14 ounces of the artificial flavors can be added.

Exercises

Solve the inequality and graph the solution on a number line:

1. $4x < 8$
2. $3x > 9$
3. $-5x \geq -10$
4. $-2x \leq -4$
5. $x - 7 \leq -6$
6. $x + 4 \geq 7$
7. $x + 2 > -1$
8. $x - 3 < -7$
9. $3 - 4x < 11$
10. $5 - 2x > 9$
11. $3x - 7 < -1$
12. $5x + 7 > -3$
13. $4x - 2 \geq x + 7$
14. $5x + 3 \leq x - 13$
15. $-2x + 2 < x - 7$
16. $2x - 3 > 4x - 7$

Solve and write the answer in set notation:

17. $2(3x - 5) > 5(2x + 4)$
18. $3(2 - 4x) \leq -2(x - 8)$

19. $7 - 3(\mathrm{x} - 3) < 6 - (\mathrm{x} - 4)$

20. $9 - 5(\mathrm{x} + 4) \geq 2 - (\mathrm{x} - 7)$

21. $6\mathrm{x} + 2(4 - 2\mathrm{x}) \geq 8 + 3(\mathrm{x} + 5)$

22. $7 - (\mathrm{x} - 3) + 2\mathrm{x} < 10 - 3(\mathrm{x} - 2)$

23. $3\mathrm{x} - [5\mathrm{x} - (-2)] \geq 2[-5 - (-2\mathrm{x}) + 3^2] - 2$

24. $5 - 3[2\mathrm{x} - (3\mathrm{x} - 2^2)] < 2^2 - 2[3\mathrm{x} - (2\mathrm{x} - 1)]$

25. $-3 < 2x - 1 \leq 5$

26. $-7 \leq 3\mathrm{x} - 4 < 8$

27. $-3 < 5x - 2 < 7$

28. $4 \leq 4\mathrm{x} - 7 \leq 8$

Use an inequality to solve the following:

29. In a math class there are five equally weighted tests. A student has grades of 94, 92, 78, and 97 on the first four tests. To earn an A in the class, the students must have an average of at least 90. What must the student get on the fifth test to earn an A in the math class?

30. For a team competition each member of a four member team runs an obstacle course. In order to pass on to the next level the team must have an average time of at most 60 seconds. If the first three member of the team run the course in 59, 68, and 53 seconds, what times for the fourth member will let the team pass onto the next level?

31. A government agency can spend at most $125,000 on a training program. If the training program has a fixed cost of $45,000 plus a cost of $125 per employee, how many employees can be trained?

32. A large company allotted at most \$75,000 for a training program. If the training program has a fixed cost of \$18,670 plus a cost of \$215 per employee, how many employees can be trained?

33. For a product to be labeled as orange juice a state agency requires that at least 45% of the drink is real orange juice. How many ounces of artificial flavors can be added to 8 ounces of real orange juice and have it still be legal to label the drink orange juice?

34. For a product to be labeled as real juice a state agency requires that at least 15% of the drink is real juice. How many ounces of artificial flavors can be added to 3 ounces of real juice and have it still be legal to label the drink as real juice?

35. The temperatures in Jerez, Spain had the following range in Celsius degrees on a summer day: $20° \leq \mathrm{C} \leq 35°$. The formula to convert Fahrenheit degrees to Celsius is $\mathrm{C} = \frac{5}{9}(\mathrm{F} - 32)$. Find the temperature range in Fahrenheit degrees for Jerez, Spain on the summer day.

75. $1.47 \text{ x } 10^{19}$ miles

Exercise Set 2.1

1. 7

3. -5

5. 57

7. $-2\frac{41}{100}$

9. -84

11. 2

13. $\frac{2}{9}$

15. $-\frac{2}{3}$

17. -45

19. 90

21. -4

23. 75

25. -18

27. a. 71.3 inches
b. The formula overestimates the actual height by 0.3 inches. This trend is not likely to continue indefinitely or men's average height would continually increases. For instance the formula indicates that the average height of men will be about eighteen and a half feet 5000 years from now.

29. a. 18,000 students
b. The formula overestimates the actual number of students by 200.

Exercise Set 2.2

1. -10xy

3. xy

5. $12 + 3x$

7. $-10y - 10$

9. $-1.55t - 8.62$

11. $5x - 7y - 2$

13. $6.34x + 2.5y - 18.11$

15. $20x - 8y + 12$

17. $-20x + 15y + 40$

19. $-2x + 7y - 8$

21. $3x - 15y$

23. $-8s - 14t$

25. $-8x^2 + 11x$

27. $9x^2 - 12x$

29. $-3x^2y + 7xy^2$

31. $-3x + 29$

33. $13x + 1$

35. $5x + 11$

37. $x - 4$

39. $11x - 29$

41. $4x^2 + 2$

43. $2x^2 - 6$

45. $4x^2 + 2x - 12$

47. $4x^2 + 2x$

49. $7x^2 - 18$

51. $-12x - 28$

53. $-20x + 64$

55. $10x + 119$

57. $4x - 47$

59. $y^2 - 9y + 21$

<u>Exercise Set 2.3</u>

1. $x = 10$

3. $x = -14$

5. $x = 1\frac{5}{12}$

7. $x = -0.12$

9. $x = -\frac{7}{8}$

11. $x = -7$

13. $x = 4$

15. $x = -3.5$

17. $x = -\frac{6}{7}$

19. $x = 4$

21. $x = -4$

23. $x = 2$

25. $x = -5$

27. $x = -16$

29. $x = -4$

31. $x = -1$

33. $x = -8$

35. $x = -1$

37. $x = 2\frac{1}{5}$

39. $x = 1\frac{5}{7}$

Exercise Set 2.4

1. $L = \frac{A}{W}$

3. $L = \frac{P-2W}{2}$.

5. $x = \frac{24+4y}{3}$

7. $y = \frac{28-4x}{-7}$.

9. $y = 3A - x - z$.

11. $r = \frac{t-Ms}{M}$

13. $C = \frac{5}{9}(F - 32)$.

15. $t = \frac{A-P}{Pr}$

Exercise Set 3.1

1. $5 + 3x$

3. $2x - 5$

5. $-5x - 5$

7. $-1.88x - 5$

9. $50 - z$

11. $40 - 2x$

13. 48

15. $1\frac{5}{12}$

17. 14 and 26

19. -32 and 25

Exercise Set 3.2

1. The odd integers are 15, 17, and 19.

3. The integers are 18, 19, and 20.

5. The stamp collector purchased thirty 5¢ stamps, ninety 15¢ stamps, and forty 25¢ stamps.

7. In 1968 the average salary for a college instructor was $10,040.

9. 35 cartridges were used over the life of the printer.

11. It will take 57 hours of training for somebody with a SAT Math score of 529 to raise their SAT Math score to 700 if we accept the SAT training program's claim.

13. After sixty hours the costs of the two plans will be the same.

15. After 280 kilowatt hours the costs of the two plans will be the same.

Exercise Set 3.3

1. 0.8 EUR : 1 USD

3. 1.24 CAD : 1 USD

5. $x = 16$

7. $x = 30$

9. $x = 12$

11. $x = 23$

13. 88.2 miles

15. There are about 245 elephants.

17. There are about 63 bears in the wildlife refuge.

19. After 19.5 more months, the employee will have accrued enough vacation days (25 days) to take a five week vacation.

21. An additional $2250 must be invested so that $450 is earned each year.

Exercise Set 3.4

1. 0.12

3. 0.0035

5. $\frac{9}{20}$

7. $\frac{143}{300}$

9. 28%

11. $55\frac{5}{9}\%$

13. $333\frac{1}{3}\%$

15. 73%

17. 16.82%

19. 1229%

21. 43.2

23. 18

25. 52%

27. 192

29. 75

31. 64%

33. There was an 18% increase.

35. There was a 15% decrease.

37. The sale price of the blender is $48.75

39. The regular price of the coffee maker is $132.

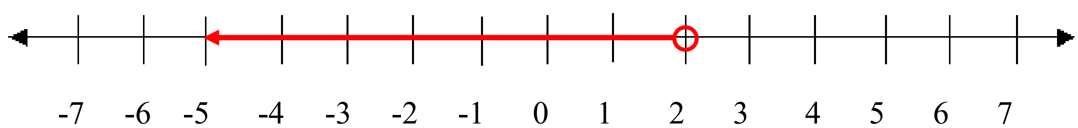
41. The value of the car after the first year is $25,004.

43. The wine shop's sale price for the French Chardonnay is $11.70.

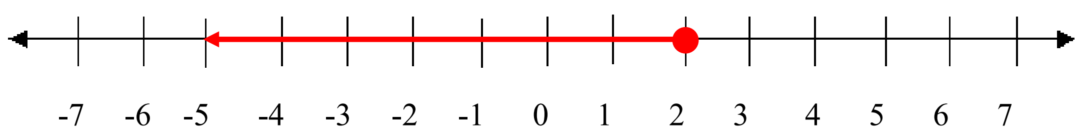
45. The supermarket's mark-up rate is 65%.

Exercise Set 3.5

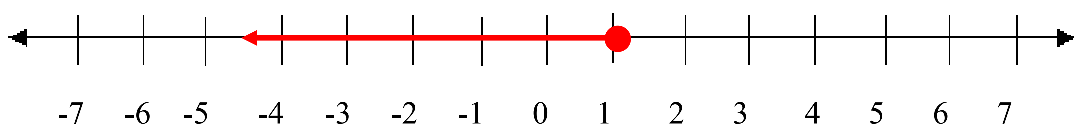
1. $x < 2$



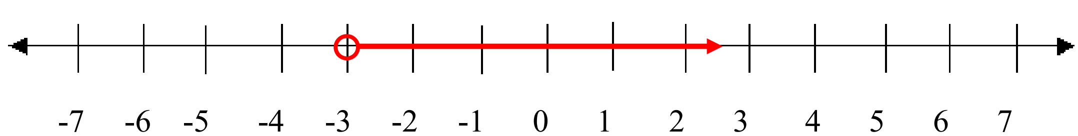
3. $x \leq 2$



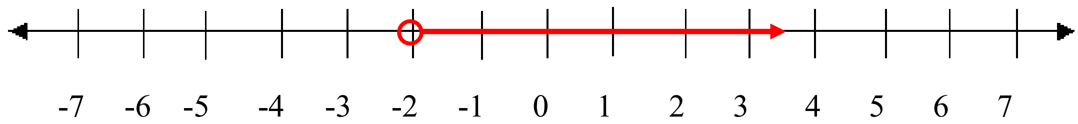
5. $x \leq 1$



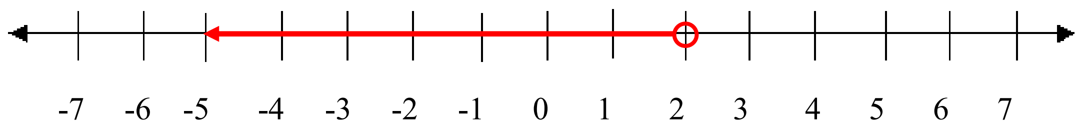
7. $x > -3$



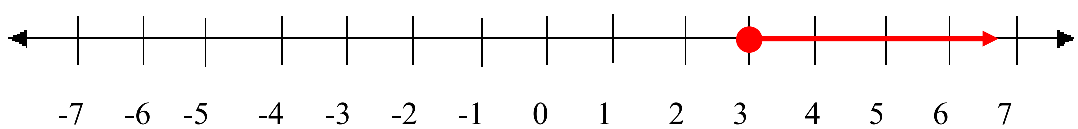
9. $x > -2$



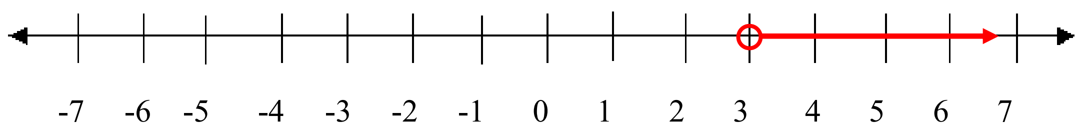
11. $x < 2$



13. $x \geq 3$



15. $x > 3$



17. $\{x | x < -7.5\}$

19. $\{x | x > 3\}$

21. $\{x | x \leq -15\}$

23. $\left\{x \middle| x \leq -\frac{4}{3}\right\}$

25. $\{x | -1 < x \leq 3\}$

27. $\left\{x \middle| -\frac{1}{5} < x < \frac{9}{5}\right\}$

29. The student needs at least an 89 on the fifth test to earn an A in the math class.

31. At most 640 employees can be trained.

33. At most $9\frac{7}{9}$ ounces of artificial flavors can be added to the real orange juice.

35. $68° \leq F \leq 95°$

Exercise Set 4.1

1. $10x^3 + 15x^2 - 20x$

3. $21x^3 - 35x^2 - 77x$

5. $-6x^4 + 9x^3 + 18x^2$

7. $-8x^4 + 20x^3 + 24x^2$

9. $15x^8 - 9x^7 - 6x^6$

11. $12x^3y - 8x^2y^2 + 20xy^3$

13. $-45x^3y^3 + 30x^4y^2 + 24x^4y^3$

15. $40x^4y^2 - 80x^3y^2 - 56x^3y^3 - 96x^4y^3$

17. $15x^2 + 22x + 8$

19. $15x^2 + 29x - 14$

21. $35x^2 - 87x + 22$

23. $42x^2 - 53x + 15$

25. $55x^2 - 74x + 24$

27. $72x^2 - 15x - 42$

29. $15x^2 + 30x - 120$

31. $6x^4 - 6x^2 - 36$

33. $72x^4 - 206x^2 + 140$

Exercise Set 4.2

1. $6x(2x^2 + x - 3)$ or $6x(2x + 3)(x - 1)$