1.1: Integers and Order of Operations

1. Define the integers
2. Graph integers on a number line.
3. Using inequality symbols < and >
4. Find the absolute value of an integer
5. Perform operations with integers
6. Use the order of operations agreement

Number Theory and Divisibility

- ***Number theory*** is primarily concerned with the properties of numbers used for counting, namely 1, 2, 3, 4, 5, and so on.
- The set of ***natural numbers*** is given by

$$N=\{1,2,3,4,5,6,7,8,9,10,11,\ldots\}$$

- Natural numbers that are multiplied together are called the ***factors*** of the resulting product.

$2\times12=24 \qquad 3\times8=24 \qquad 4\times6=24$

Therefore, the factors of 24 are:
1, 2, 3, 4, 6, 8, 12, 24

Define the Integers

- The set consisting of the natural numbers, 0, and the negatives of the natural numbers is called the set of *integers*.

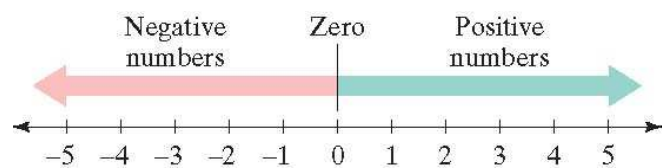
$$\text{Integers} = \{\underbrace{\ldots, -4, -3, -2, -1}_{\text{Negative integers}}, 0, \underbrace{1, 2, 3, 4, \ldots}_{\text{Positive integers}}\}$$

Notice the term *positive integers* is another name for the *natural numbers*. The positive integers can be written in two ways:

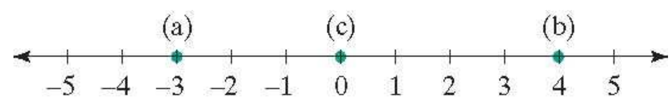
1. Use a "+" sign. For example, +4 is "positive four".
2. Do not write any sign. For example, 4 is also "positive four".

The Number Line

- The ***number line*** is a graph we use to visualize the set of integers, as well as sets of other numbers.
- Notice, zero is neither positive nor negative.



- **Graph:** **a.** –3 **b.** 4 **c.** 0



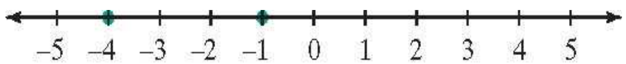
Inequality Symbols < >

- These symbols always point to the lesser of the two real numbers when the inequality statement is true.

$-4 < -1$ — −4 is less than −1 because −4 is to the **left** of −1 on the number line.

$-1 > -4$ — −1 is greater than −4 because −1 is to the **right** of −4 on the number line.

- Looking at the graph, –4 and –1 are graphed below.



- **Exercise:** –4 ▢ 3 –1 ▢ –5 –5 ▢ –2 0 ▢ –3

Addition of Integers

Rule

If integers have same sign,
1. Add their absolute values
2. The sign of the sum is the same sign of the two numbers.

Examples

$-11 + (-15) = -26$ — Add absolute values: $11 + 15 = 26$. Use the common sign.

$-13 + 4 = -9$ — Subtract absolute values: $13 - 4 = 9$. Use the sign of the number with the greater absolute value.

If integers have different signs,
1. Subtract the smaller absolute value from the larger absolute value.
2. The sign of the sum is the same as the sign of the number with the larger absolute value.

$13 + (-6) = 7$ — Subtract absolute values: $13 - 6 = 7$. Use the sign of the number with the greater absolute value.

A good analogy for adding integers is temperatures above and below zero on the thermometer. Think of a thermometer as a number line standing straight up.

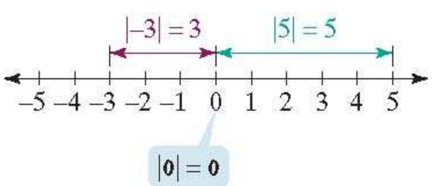
Absolute Value

- The *absolute value* of an integer *a*, denoted by |*a*|, is distance from 0 to *a* on number line
- Because absolute value describes a distance, it is never negative

Example: Find the absolute value:

a. |–3| b. |5| c. |0|

Solution:



Subtraction of Integers

- *Additive inverses* have same absolute value, but lie opposite sides of zero on number line
  - When we add additive inverses, the sum is equal to zero. $a + (-a) = 0$
  - For example: $18 + (-18) = 0$ $(-7) + 7 = 0$
  - Sum of any integer and its additive inverse = 0
- For all integers *a* and *b*,

$$a - b = a + (-b)$$

- In words, to subtract *b* from *a*, add the *additive inverse* of *b* to *a*
- The result of subtraction is called the *difference*

Example: Subtracting Integers

Subtract:

a. **17 – (–11)** b. **–18 – (–5)** c. **–18 – 5**

a. $17 - (-11) = 17 + 11 = 28$

Change the subtraction to addition. | Replace –11 with its additive inverse.

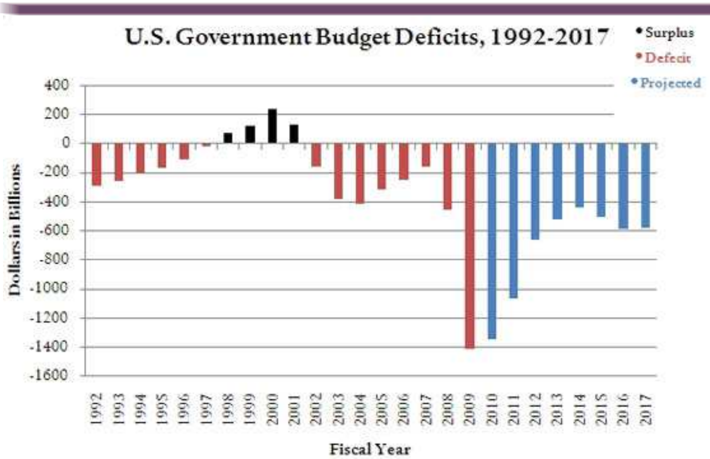
b. $-18 - (-5) = -18 + 5 = -13$

Change the subtraction to addition. | Replace –5 with its additive inverse.

c. $-18 - 5 = -18 + (-5) = -23$

Change the subtraction to addition. | Replace 5 with its additive inverse.

What is difference between 2000 and 2008?



Multiplication of Integers

- The result of multiplying two or more numbers is called the ***product***
- **Rules**
  1. The product of two integers with <u>different signs</u> is found by multiplying their absolute values. The <u>product is negative</u>. $7(-5) = -35$
  2. The product of two integers with the <u>same signs</u> is found by multiplying their absolute values. The <u>product is positive</u>. $(-6)(-11) = 66$
  3. The product of <u>0 and any integer</u> is <u>0</u>. $-17(0) = 0$
  4. Product of an odd number of negative factors is negative. $(-6)(-5)(-3) = -90$
  5. Product of an even number of negative factors is positive. $(-6)(5)(-3)(2) = 180$

Division of Integers

- The result of dividing the integer a by the integer b is called the ***quotient***
- We write this quotient as: $a \div b$ or a / b or $\frac{a}{b}$
- **Rules**
  - Quotient of two integers with <u>different signs</u> is found by dividing their absolute values. <u>Quotient is negative</u>. $\frac{80}{-4} = \frac{-80}{4} = -\frac{80}{4} = -20$
  - The quotient of two integers with <u>same sign</u> is found by dividing their absolute values. <u>Quotient is positive</u>. $\frac{27}{9} = 3$ $\frac{-45}{-3} = 15$
  - Zero divided by any nonzero integer is zero. $\frac{0}{-5} = 0$
  - Division by 0 is undefined or infinity. $\frac{-5}{0} = \infty = \text{undefined}$

Exponential Notation

- Because exponents indicate repeated multiplication, rules for multiplying can be used to evaluate exponential expressions.

Evaluate: a. $(-6)^2$ b. -6^2 c. $(-5)^3$ d. $(-2)^4$

Solution:

a. $(-6^2)=(-6)\cdot(-6)=36$

b. $-6^2=-\{(6)\cdot(6)\}=(-1)\cdot(6)\cdot(6)=36$

c. $(-5)^3=(-5)\cdot(-5)\cdot(-5)=-125$

d. $(-2)^4=(-2)\cdot(-2)\cdot(-2)\cdot(-2)=16$

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Simplify Each Expression

$$7^2-48\div 4^2\cdot 5+2$$

$$(-8)^2-(10-13)^2(-2)$$

$$-4\{2[-3+(-7)]-4[2^3+4(-2)^3]\}$$

$$\frac{12\div 3\cdot 5(2^2+3^2)}{7+3-6^2}$$

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Order of Operations = PEMDAS

1. Perform all operations within grouping symbols Parenthesis ( ) { } [ ]
2. Evaluate all Exponential expressions.
3. Do all the Multiplications and Divisions in the order in which they occur, from left to right.
4. Finally, do all Additions and Subtractions in order in which they occur, from left to right.

Simplify: $6^2 - 24 \div 2^2 \cdot 3 + 1$

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1.2: Prime Factorization, GCF, LCM

1. Determine *divisibility*
2. Write the *Prime Factorization* of a *Composite Number*
3. Find the *Greatest Common Factor* GCF of two numbers
4. Solve problems using the GCF or *Greatest Common Factor*
5. Find the *Least Common Multiple* LCM of two numbers
6. Solve problems using the LCM or *Least Common Multiple*

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Divisibility

- If *a* and *b* are natural numbers, *a* is ***divisible*** by *b* if the operation of dividing *a* by *b* leaves a remainder of 0
  - Divisibility by 2 = Last digit is even 0, 2, 4, 6, 8
  - Divisibility by 3 = Sum of digits is divisible by 3
  - Divisibility by 5 = Last digit is 0 or 5
  - Divisibility by 10 = Last digit is 0
  - Other divisibility checks can be done on calculator using division
    - Number is divisible if there is no digits to right of decimal
    - There is no remainder

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Prime Factorization

- A ***prime number*** is a natural number greater than 1 that has only itself and 1 as factors
- A ***composite number*** is a natural number greater than 1 that is divisible by a number other than itself and 1
- The Fundamental Theorem of Arithmetic

  Every composite number can be expressed as a product of prime numbers in one and only one way
- Method used to find the ***Prime Factorization*** of a composite number is called a ***factor tree***

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Example: Prime Factorization using a Factor Tree

Exercise: Find the prime factorization of 700.

Solution: Do successive division of prime numbers beginning with 2 and incrementing to next prime number 3, 5, 7, 11…

700 → 350 → 175 → 35 → 7

2 2 5 5

Thus, the prime factorization of 700 is

$700 = 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7$

$= 2^2 \cdot 5^2 \cdot 7$

Arrange the factors from least to greatest as shown

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Greatest Common Factor

1. Write the prime factorization of each number
2. Select each prime factor with the smallest exponent that is common to each of the prime factorizations
3. Form the product of the numbers from step 2. The Greatest Common Factor is the product of the factors

Exercise: Find the Greatest Common Factor of 216 and 234

216 → 108 → 54 → 27 → 9 → 3

2 2 2 3 3

234 → 117 → 39 → 13

2 3 3

Prime factorizations are:

$216 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

$= 2^3 \cdot 3^3$

$234 = 2 \cdot 3 \cdot 3 \cdot 13$

$= 2 \cdot 3^2 \cdot 13$

Greatest Common Factor

$GCF = 2 \cdot 3^2 = 2 \cdot 9 = 18$

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Example : Word Problem Using the GCF

Exercise: A sports league is created by dividing a group of 40 men and 24 women into all-male and all-female teams so that each team has the same number of people.
What is the largest number of people that can be placed on a team?

Solution: We are looking for GCF of 40 and 24

Begin with the prime factorization of 40 and 24:

$$40 = 2^3 \cdot 5$$
$$24 = 2^3 \cdot 3$$

Now select common prime factors, with lowest exponent

$$GCF = 2^3 = 8$$

Therefore, each team should be comprised of 8 team members with 5 males teams and 3 females teams

Example: Finding Least Common Multiple

Exercise: Find the least common multiple of 144 and 300

Solution: Step 1. Write the prime factorization of numbers

$$144 = 2^4 \cdot 3^2$$
$$300 = 2^2 \cdot 3 \cdot 5^2$$

Step 2. Select every prime factor that occurs, raised to the greatest power to which it occurs, in these factorizations.

$$144 = 2^4 \cdot 3^2$$
$$300 = 2^2 \cdot 3 \cdot 5^2$$

Step 3. Form the product of the numbers from step 2. The least common multiple is the product of these factors.

$$LCM = 2^4 \cdot 3^2 \cdot 5^2 = 16 \cdot 9 \cdot 25 = 3600$$

Hence, the LCM of 144 and 300 is 3600.
Thus, the smallest natural number divisible by 144 and 300 is 3600

Least Common Multiple

- The **LCM** of two natural numbers is the smallest natural number that is divisible by all of the numbers

1. Write the prime factorization of each number
2. Select every prime factor that occurs, raised to the greatest power to which it occurs, in these factorizations
3. Form the product of the numbers from step 2. The least common multiple is the product of these factors

Example : Word Problem Using the LCM

Exercise: A movie theater runs its films continuously. One movie runs for 80 minutes and a second runs for 120 minutes. Both movies begin at 4:00 P.M. When will the movies begin again at the same time?

Solution: We are looking for LCM of 80 and 120.
Find LCM and add this number of minutes to 4:00 P.M.

Begin with the prime factorization of 80 and 120:

$$80 = 2^4 \cdot 5$$
$$120 = 2^3 \cdot 3 \cdot 5$$

Now select each prime factor, with the greatest exponent

$$LCM = 2^4 \cdot 3 \cdot 5 = 16 \cdot 3 \cdot 5 = 240$$

Therefore, it will take 240 minutes, or 4 hours, for the movies to begin again at the same time at 8:00 P.M.